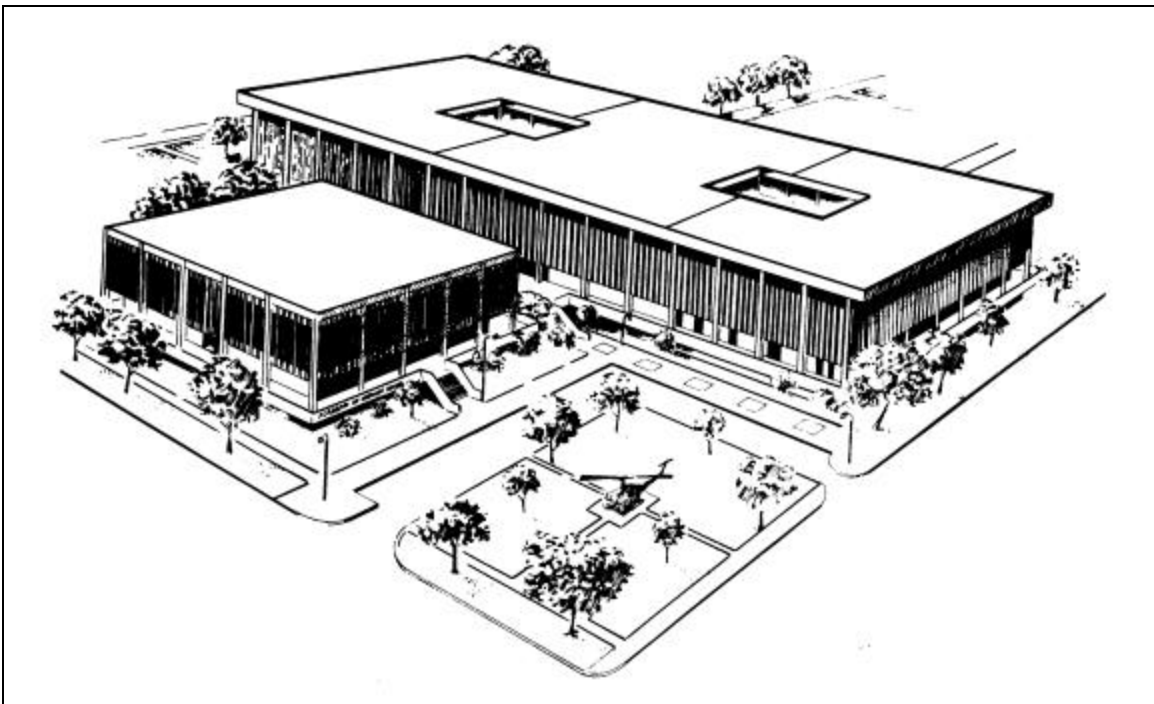


# MD0900

UNITED STATES ARMY  
MEDICAL DEPARTMENT CENTER AND  
SCHOOL  
FORT SAM HOUSTON, TEXAS 78234



## BASIC MATHEMATICS

Edition 100

## **DEVELOPMENT**

This subcourse reflects the current thought of the Academy of Health Sciences and conforms to printed Department of the Army doctrine as closely as currently possible. Development and progress render such doctrine continuously subject to change.

When used in this publication, words such as "he," "him," "his," and "men" are intended to include both the masculine and feminine genders, unless specifically stated otherwise or when obvious in context.

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## **ADMINISTRATION**

Students who desire credit hours for this correspondence subcourse must meet eligibility requirements and must enroll through the Nonresident Instruction Branch of the U.S. Army Medical Department Center and School (AMEDDC&S).

Application for enrollment should be made at the Internet website: <http://www.atrrs.army.mil>. You can access the course catalog in the upper right corner. Enter School Code 555 for medical correspondence courses. Copy down the course number and title. To apply for enrollment, return to the main ATRRS screen and scroll down the right side for ATRRS Channels. Click on SELF DEVELOPMENT to open the application and then follow the on screen instructions.

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## TABLE OF CONTENTS

<u>Lesson</u>	<u>Frames</u>	<u>Page</u>
INTRODUCTION.....		iii
<b>1 REVIEW OF WHOLE NUMBERS</b>		
Place Value .....	1-1 -- 1-2	1-1
Parts of a Problem .....	1-3 -- 1-8	1-2
Addition .....	1-9 -- 1-11	1-4
Subtraction .....	1-12 -- 1-15	1-5
Multiplication .....	1-16 -- 1-19	1-7
Division .....	1-20 -- 1-25	1-8
Checking .....	1-26 -- 1-32	1-11
Combined Operations .....	1-33 -- 1-35	1-14
Self-Test .....	1-36	1-16
<b>2 FRACTIONS</b>		
Definition .....	2-1 -- 2-2	2-1
Multiplying and Dividing Fractions .....	2-3 -- 2-6	2-2
Adding and Subtracting Fractions .....	2-7 -- 2-24	2-3
Reducing Fractions .....	2-26 -- 2-30	2-11
Improper Fractions .....	2-31 -- 2-37	2-14
Self-Test .....	2-38	2-17
<b>3 DECIMALS</b>		
Definition .....	3-1	3-2
Reading and Writing Decimals .....	3-2 -- 3-7	3-2
Changing Fractions to Decimals .....	3-8 -- 3-11	3-5
Changing Decimals to Fractions .....	3-12 -- 3-12	3-6
Adding Decimals .....	3-13	3-7
Subtracting Decimals .....	3-14	3-7
Multiplying Decimals .....	3-15 -- 3-17	3-8
Dividing Decimals .....	3-18 -- 3-20	3-9
"Rounding" Decimals .....	3-21 -- 3-28	3-10
Percents .....	3-29 -- 3-35	3-17
Self-Test .....	3-36	3-21

<u>Lesson</u>		<u>Frames</u>	<u>Page</u>
4	THE METRIC SYSTEM		
	History .....	4-1 -- 4-3	4-1
	Basic Metric Measures .....	4-4 -- 4-8	4-3
	Prefixes and Root Words .....	4-9 -- 4-14	4-5
	Units of Distance .....	4-15 -- 4-20	4-7
	Units of Volume .....	4-21 -- 4-31	4-10
	Units of Mass/Weight .....	4-32 -- 4-35	4-15
	Units of Area .....	4-36 -- 4-39	4-18
	Temperature .....	4-40 -- 4-48	4-20
	Self-Test .....	4-49	4-25
5	NEGATIVE NUMBERS, SCIENTIFIC NOTATION, AND SQUARE ROOTS		
	Negative Numbers .....	5-1 -- 5-12	5-1
	Scientific Notation .....	5-13 -- 5-23	5-8
	Squares and Square Roots .....	5-24 -- 5-32	5-14
	Self-Test .....	5-33	5-21
	APPENDIX .....		A-1

**CORRESPONDENCE COURSE OF  
U.S. ARMY MEDICAL DEPARTMENT CENTER AND SCHOOL**

**SUBCOURSE MD0900**

**BASIC MATHEMATICS**

**INTRODUCTION**

This programmed booklet is designed to help you to correctly perform basic mathematical operations involving addition, subtraction, multiplication, and division of whole numbers, decimals, fractions, and negative numbers. The subcourse also presents instruction for working with metric (SI) units and for converting metric units to the U.S. Customary System and for converting U.S. Customary System units to metric units.

This text consists of five lessons and a final examination:

Lesson 1, Review of Whole Numbers

Lesson 2, Fractions

Lesson 3, Decimals

Lesson 4, The Metric System

Lesson 5, Negative Numbers, Scientific Notation, and Square Roots

You will be awarded 8 credit hours for the successful completion of this subcourse.

In addition to this subcourse, you are furnished an examination answer sheet and an envelope. You must furnish a #2 pencil for marking your examination answer sheet.

You are encouraged to complete the subcourse lesson by lesson. You are also encouraged to work the self-test found in the last frame of each lesson. Working the self-test and the exercises in the other frames will help you to accomplish the lesson objectives and to prepare you for the final examination.

Use the enclosed envelope to submit your completed examination answer sheet to the U.S. Army Medical Department Center and School (AMEDDC&S) for grading. You will be notified by return mail of the results.

We suggest that you follow these study procedures:

Read and work each lesson frame carefully. Check your answer against the solution given in the second column of the following frame. [SUGGESTION: Use a sheet of paper to cover the solution while working the problem.]

Work the self-test exercises at the end of the first lesson. Refer to the lesson frames as needed. When you have completed the exercises to your satisfaction, compare your answers with the solutions following the self-text frame. For each exercise answered incorrectly, review the appropriate lesson frame(s) and then rework the problem.

After you have successfully completed one lesson, go to the next and repeat the above procedures.

When you feel confident that you have mastered the study materials, complete the examination. We suggest that you work the examination by first marking your answers in the subcourse booklet. When you have completed the examination items to your satisfaction, transfer your responses to the examination answer sheet and mail it to the AMEDDC&S for grading.

The grade you make on the examination will be your rating for the subcourse.

A Student Comment Sheet is located at the back of this subcourse. Please enter any suggestions or comments that will help us to improve the subcourse. If you complete the comment sheet, please include it in the envelope when you submit your examination answer sheet for grading.

If you wish a reply, please send a letter with your name, rank, social security number, and return address along with your question. Please state the subcourse number and edition along with the frame or examination item about which you have a question.

### Terminal Learning Objectives

At the completion of this subcourse, you will be able to correctly perform basic mathematical operations involving addition, subtraction, multiplication, and division of whole numbers, fractions, decimals, and negative numbers.

At the completion of this subcourse, you will be able to correctly perform basic operations involving the metric (SI) system, including conversion between the metric system and the U.S. Customary System.

## Presentation

This subcourse uses the technique of the programmed instruction. Basically, programmed instruction presents information in small bits called "frames." A frame usually requires you to use the information to answer a question or solve a problem. Feedback is usually provided in the second column (shaded text) of the following frame. Each frame is numbered. You should proceed in numerical order.

At the beginning of each lesson, you will find a list of objectives. These lesson objectives state what you are expected to learn by the end of the lesson. Read them carefully before beginning the lesson frames.

## LESSON ASSIGNMENT

<b>LESSON 1</b>	Review of Whole Numbers.
<b>LESSON ASSIGNMENT</b>	Frames 1-1 through 1-36.
<b>MATERIALS REQUIRED</b>	Pencil, eraser.
<b>LESSON OBJECTIVES</b>	After completing this lesson, you should be able to:  1-1. Identify by name each number in given addition, subtraction, multiplication, and division problems.  1-2. Set up and solve given problems involving addition, subtraction, multiplication, and division of whole numbers.  1-3. Check given problems involving addition, subtraction, multiplication, and division of whole numbers.
<b>SUGGESTIONS</b>	Work the following exercises (numbered frames) in numerical order. Write the answer in the space provided in the frame. After you have completed a frame, check your answer against solution given in the shaded area in the following frame. The final frame contains review exercises for Lesson 1. These exercises will help you to achieve the lesson objectives and prepare for the examination.

---

### FRAME 1-1.

**PLACE VALUE.** Our number system is based upon powers of 10. That is, the value (amount) of a digit (numeral) depends upon its location in the number. Consider a given digit location (its place in the number, not the value of the numeral itself). The digit location to its immediate left is worth ten times as much as the given digit location. The digit place to the immediate right is worth one-tenth as much. This is called place value. For example, in the number 456, the "5" tells how many tens (place value is "10"), the "4" tells how many hundreds (place value is 100, which is  $10 \times 10$ ) and the 6 tells how many ones (place value is 1, which is  $1/10 \times 10$ ). This is sometimes called "the base 10 numbering system."

The number 456 is equal to

	4 x 100
	plus 5 x 10
	plus 6 x 1

In the number 9724, the digit in the far right tells how many ones ( $4 \times 1$ ). The second digit tells how many tens ( $2 \times 10$ ). The third digit tells how many hundreds ( $7 \times 100$ ). The fourth digit tells how many \_\_\_\_\_ ( $9 \times$  \_\_\_\_\_).

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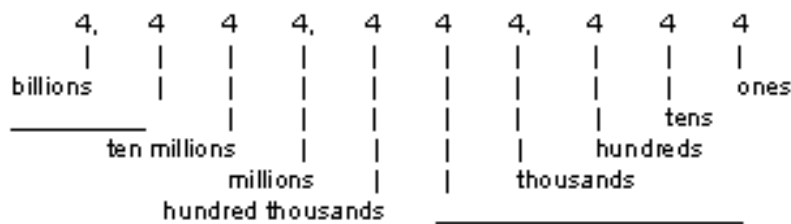
The solution to the exercise in Frame 1-1 is in the shaded area (right side) of Frame 1-2 on the following page.

**FRAME 1-2.**

Remember: When dealing with whole numbers (no fractions or decimals), the numeral to the far right tells how many ones (1), the numeral to its left tells how many tens (10 x 1), the next numeral to the left tells how many hundreds (10 x 10), the next numeral tells how many thousands (10 x 100), and so on with the place value increasing by a factor of ten each time.

**NOTE:** In the above statement, number refers to the entire value. Numeral refers to one digit (symbol) within the number.

Test your understanding of place values by filling in the blanks below.



Solution to  
Frame 1-1.

thousands

1000

**FRAME 1-3.**

**PARTS OF A PROBLEM.** Just like members of a family can be identified by their relationship to other members (mother, son, aunt, brother, etc.), the different numbers in a math problem can also be identified by their relationships. For example, the number that results when two or more numbers are added together is called the SUM. The numbers that are added together are called ADDENDS.

Label the parts of the problem below :

$$\begin{array}{r}
 4444 \quad \underline{\hspace{2cm}} \\
 + 333 \quad \underline{\hspace{2cm}} \\
 \hline
 4777 \quad \underline{\hspace{2cm}}
 \end{array}$$

Solution to  
Frame 1-2.

4,444,44,444

hundred millions  
ten thousands

**FRAME 1-4.**

The parts in a subtraction problem are the minuend, subtrahend, and the remainder. The remainder is sometimes called the "difference."

Example:        978    minuend  
                  -243   subtrahend  
                  735    remainder (or difference)

Solution to  
Frame 1-3.

addend  
+ addend  
sum

The answer in a subtraction problem is called the \_\_\_\_\_.  
The top number is the \_\_\_\_\_.  
The number subtracted from the top number is called the \_\_\_\_\_.

---

**FRAME 1-5.**

A multiplication problems consist of a multiplicand, a multiplier, and a product. The multiplicand (top number) is the number to be multiplied. The multiplier (second number) is the number doing the multiplying. The answer is called the product. The numbers being multiplied together (the multiplicand and the multiplier) are sometimes referred to as "factors."

Label the parts of the following multiplication problem.

45 \_\_\_\_\_  
x 4 \_\_\_\_\_  
180 \_\_\_\_\_

Solution to  
Frame 1-4

remainder (or difference)  
minuend  
subtrahend

In this problem, the "45" and "4" can also be called \_\_\_\_\_.

---

**FRAME 1-6.**

Division is used to determine the number of times one number is contained in another number. If you were to divide 18 by 6, you might ask yourself, "How many groups of 6's are there in 18?" Your answer would be 3. The answer is called the quotient. The number that is being divided is called the dividend. The divisor is the number that is divided into the dividend.

Solution to  
Frame 1-5

      multiplicand  
x multiplier  
      product  
  
factors

In the problem  $18 \div 6 = 3$  (18 divided by 6 equals 3),

"6" is the \_\_\_\_\_,

"18" is the \_\_\_\_\_,

and "3" is the \_\_\_\_\_.

---

**FRAME 1-7.**

If the divisor does not go into the dividend evenly, an amount is left over. The quantity left is called the remainder. For example, when the number 19 is divided by 6, the quotient is 3 with a remainder of 1. (There are three groups of sixes and another group consisting of the one.) The remainder is usually expressed as a fraction (remainder over divisor).

Solution to  
Frame 1-6

6 - divisor  
18 - dividend  
3 - quotient

Label the parts of the problem  $442 \div 15$  below.

$\begin{array}{r} \underline{29} \\ 15 \overline{) 442} \\ \underline{30} \\ 142 \\ \underline{135} \\ 7 \end{array}$	$\begin{array}{l} 442 \text{ _____} \\ 15 \text{ _____} \\ 29 \text{ _____} \\ 7 \text{ _____} \end{array}$
---	---

**FRAME 1-8.**

The answer to the problem  $19 \div 6$  can be written as 3r1 (three with a remainder of 1) or as  $3 \frac{1}{6}$  (three and one-sixth). The second method (whole number and a fraction) is the preferred method of stating the answer. Another way of expressing the answer (especially if you are using a calculator) is as a decimal (see Frame 1-24).

Solution to  
Frame 1-7

442 - dividend  
15 - divisor  
29 - quotient  
7 - remainder

The answer to the problem  $442 \div 15$  can be written as

\_\_\_\_\_ or as \_\_\_\_\_.

**FRAME 1-9.**

**ADDITION.** Any problem in addition, subtraction, multiplication, or division must be set up correctly in order to solve it. In addition, you must put like units under like units (ones under ones, tens under tens, hundreds under hundreds, etc.).

Solution to  
Frame 1-8

29r7 (29 with a remainder of 7) or  $29 \frac{7}{15}$  (twenty-nine and seven-fifteenths.)

Set up and work this addition problem:  $3 + 212 + 21 = ?$

**FRAME 1-10.**

In the previous addition problem, none of the columns added up to more than 9. If the sum of a column is more than 9, write down the last (right) digit under the column and add the remaining digit(s) to the next column (the column to the left). **REMEMBER:** When adding, begin with the column on the far right and then go to the left.

$\begin{array}{r} 1 \\ 326 \\ + 146 \\ \hline 472 \end{array}$	<p>In the first (right) column, <math>6 + 6 = 12</math>. The "2" is written below the column and the "1" is <u>carried</u> to the next column where it is added with the "2" and the "4." (The "1," "2," and "4" are all <u>tens</u>.)</p>
--	--

A more complicated addition problem is shown below.

$\begin{array}{r} 1 \ 1 \ 2 \\ 367 \\ 1,416 \\ 6,576 \\ \hline 2,108 \\ 10,467 \end{array}$	<p><math>7 + 6 + 6 + 8 = 27</math> Write "7" carry "2"  <math>2 + 6 + 1 + 7 + 0 = 16</math> Write "6" carry "1"  <math>1 + 3 + 4 + 5 + 1 = 14</math> Write "4" carry "1"  <math>1 + 0 + 1 + 6 + 2 = 10</math> Write "10" (There is no column to the left.)</p>
---	--

**NOTE:** You do not have to write the commas when adding, but may help to keep the columns straight.

Now you add these numbers: "1459," "38," and "327."

Solution to Frame 1-9

$\begin{array}{r} 3 \\ 212 \\ \hline 236 \end{array}$	<p>or</p> $\begin{array}{r} 003 \\ 212 \\ \hline 021 \\ 236 \end{array}$
---	--

3 ones + 2 ones + 1 one = 6 ones.

0 tens + 1 ten + 2 tens = 3 tens.

0 hundreds + 2 hundreds + 0 hundreds = 3 hundreds.

**FRAME 1-11**

Another way of thinking about the problem  $367 + 1416 + 6576 + 2108$  is

$7 +$	$6 +$	$6 +$	$8 =$	$27$
$60 +$	$10 +$	$70 +$	$00 =$	$140$
$300 +$	$400 +$	$500 +$	$100 =$	$1300$
$0000 +$	$1000 +$	$6000 +$	$2000 =$	$\underline{9000}$
				$10467$

Solution to Frame 1-10

$$\begin{array}{r} 1459 \\ 38 \\ \hline 327 \\ 1824 \end{array}$$

**FRAME 1-12.**

**SUBTRACTION.** Now, let's subtract whole numbers. Just as in addition, like units must go under like units (ones under ones, tens under tens, etc.).

Set up and work this subtraction problem in the space to the right:  $3697 - 375$

Solution to Frame 1-11

No problem was given.

**FRAME 1-13.**

If the top numeral in the column is smaller than the numeral beneath it, then you must "borrow 10" from the top numeral in the column to the left. (Remember that each numeral in the left column has a place value that is ten times greater. Therefore, to "borrow 10," decrease the numeral in the left column by "1" and add "10" to the top numeral in the column with which you are working. In the problem "43 minus 17," four tens become three tens and three ones becomes 13 ones.) This problem is worked below. Another (longer) procedure for performing the same operation is shown to the right.

$$\begin{array}{r}
 \overset{3}{4} \overset{13}{3} \\
 \underline{-17} \\
 26
 \end{array}
 \quad
 43 = 40 + 3 = 30 + 13 = 30 \quad 13$$

$$\begin{array}{r}
 \overset{3}{4} \overset{13}{3} \\
 \underline{-17} \quad \underline{-(10+7)} \quad \underline{-(10+7)} \quad \underline{-10} \quad \underline{-7} \\
 20 + 6 = 26
 \end{array}$$

Work this problem using the "borrowing" method: 542 minus 264.

NOTE: You will have to borrow more than one time.

Solution to  
Frame 1-12

$$\begin{array}{r}
 3697 \\
 \underline{-375} \\
 3322
 \end{array}
 \quad
 \text{OR}
 \quad
 \begin{array}{r}
 3697 \\
 \underline{-0375} \\
 3322
 \end{array}$$

OR

$$\begin{array}{l}
 0007 - 0005 = 0002 \\
 0090 - 0070 = 0020 \\
 0600 - 0300 = 0300 \\
 \underline{3000 - 0000} = 3000 \\
 3697 - 0375 = 3322
 \end{array}$$

**FRAME 1-14.**

Check your understanding of the borrowing method by working the same problem using the longer method shown in Frame 1-13.

Solution to  
Frame 1-13

$$\begin{array}{r}
 \overset{4}{5} \overset{13}{4} \overset{12}{2} \\
 \underline{-264} \\
 278
 \end{array}$$

**FRAME 1-15.**

What happens if you need to borrow from the column on the left, but there is a zero in that column? You must go one column more to the left, "borrow 10" in order to change the zero into "10," and then borrow from the "10." For example, subtract "7" from "403."

$$\begin{array}{r}
 \overset{3}{4} \overset{9}{0} \overset{13}{3} \\
 \underline{-7} \\
 396
 \end{array}
 \quad
 403 = 400 + 00 + 3 = 300 + 100 + 3 = 300 + 90 + 13$$

$$\begin{array}{r}
 \overset{3}{4} \overset{9}{0} \overset{13}{3} \\
 \underline{-7} \quad \underline{-7} \quad \underline{-7} \quad \underline{-000} \quad \underline{-00} \quad \underline{-7} \\
 300 + 90 + 6 = 396
 \end{array}$$

Now you solve this one:  $\begin{array}{r} 5003 \\ \underline{-1009} \end{array}$

Solution to  
Frame 1-14

$$\begin{array}{l}
 500 + 40 + 2 \\
 \underline{-(200 + 60 + 4)}
 \end{array}$$

$$\begin{array}{l}
 500 + 30 + 12 \\
 \underline{-(200 + 60 + 4)}
 \end{array}$$

$$\begin{array}{l}
 400 + 130 + 12 \\
 \underline{-200} \quad \underline{-60} \quad \underline{-4} \\
 200 + 070 + 08 = 278
 \end{array}$$

**FRAME 1-16.**

**MULTIPLICATION.** Multiplication is actually a shortened form of addition. For example:

$$9 \times 4 = 9 + 9 + 9 + 9 = 36.$$

Also note that  $9 \times 4 = 4 \times 9 = 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 36$ .

**NOTE:** When a multiplication problem is set up, the number with the most digits is usually chosen to be the multiplicand (the number on top).

The problem  $367 \times 97$  can also be solved by writing 367 down \_\_\_\_\_ times and adding the numbers together or by writing 97 down \_\_\_\_\_ times and adding the numbers together.

Solution to  
Frame 1-15

$$\begin{array}{r} 49913 \\ 5003 \\ -1009 \\ \hline 3994 \end{array}$$

$$\begin{array}{r} 4000+900+90+13 \\ -1000-000-00-9 \\ \hline 3000+900+90+4 \end{array}$$

**FRAME 1-17.**

Like addition, you need to carry when the product is more than nine.

**Remember to add the number you carried after you multiply.** For example:  $24 \times 3$ .

$$\begin{array}{r} 1 \\ 24 \\ \times 3 \\ \hline 72 \end{array} \quad \begin{array}{l} 4 \times 3 = 12. \text{ Write the "2" and carry the 1 (really 10), then add} \\ 2 \times 3 = 6 \text{ the number carried (1) to the product (6)} \\ \text{[really six 10's]. } 6 + 1 = 7. \text{ Write down the "7".} \end{array}$$

Another way of stating the problem is:

$$\begin{array}{r} 24 = 20 + 4 \\ \times 3 \quad \times 3 \quad \times 3 \\ 60 + 12 = 72 \end{array}$$

Now you work this problem:  $3,415 \times 4$

Solution to  
Frame 1-16

$$97$$

$$367$$

(You can see that multiplication takes less time and results in fewer math errors.)

**FRAME 1-18.**

When the multiplier has more than one digit, the multiplicand is multiplied by each digit in the multiplier beginning with the right digit of the multiplier. The products are added together to obtain the final answer. Note that the right digit of the product is in the same column as the digit of the multiplier being used. You may wish to fill in the empty digit places in the product with zeros to help keep the columns straight. For example:  $621 \times 27$ .

$$\begin{array}{r} 1 \\ 621 \\ \times 27 \\ \hline 4347 \\ 12420 \\ \hline 16767 \end{array} \quad \begin{array}{r} 1 \\ 621 \\ \times 27 \\ \hline 4347 \\ 12420 \\ \hline 16767 \end{array} \quad \begin{array}{r} 1 \\ 621 \\ \times 27 \\ \hline 4347 \\ 12420 \\ \hline 16767 \end{array} \quad \begin{array}{r} 1 \\ 621 \\ \times 27 \\ \hline 4347 \\ 12420 \\ \hline 16767 \end{array} \quad \begin{array}{r} 1 \\ 621 \\ \times 27 \\ \hline 4347 \\ 12420 \\ \hline 16767 \end{array}$$

You work this problem:  $367 \times 97$

Solution to  
Frame 1-17

$$\begin{array}{r} 1 \quad 2 \\ 3415 \\ \times 4 \\ \hline 13660 \end{array}$$

$$\begin{array}{r} 5 \times 4 = 20 \\ 10 \times 4 = 40 \\ 400 \times 4 = 1,600 \\ 3000 \times 4 = 12,000 \\ \hline 13,660 \end{array}$$

**FRAME 1-19.**

Normally, each group of three digits (beginning at the decimal point and going to the left) is set off by a comma in order to make reading easier (separate thousands from hundreds, millions from thousands, billions from millions, etc.).

If the product has only four digits (less than 10,000), the comma may be present or absent (4,500 or 4500). If the number has five or more digits, include the comma(s) in the product. Numbers of three or less digits (less than 1,000) have no commas.

**NOTE:** If the number is a decimal number (discussed in Lesson 3), the above comments apply only to digits to the left of the decimal point. Commas are not placed between digits located to the right of the decimal point.

Remember to keep your columns straight. This is especially true if the multiplier contains a zero. Also remember that the product of any number multiplied by zero is zero.

**NOTE:** You may wish to remove the commas from the factors when multiplying if that will help you to keep the columns straighter and keep you from becoming confused.

Work this problem: 23,042 x 1,020

Solution to  
Frame 1-18

$$\begin{array}{r}
 66 \\
 44 \\
 367 \\
 \times 97 \\
 \hline
 2569 \quad (367 \times 7) \\
 33030 \quad (367 \times 90) \\
 \hline
 35,599
 \end{array}$$

**FRAME 1-20.**

**DIVISION.** The division problem "250 ÷ 25 = 10" (which is read "250 divided by 25 equals 10") means that 250 contains 10 sets (groups) of 25.

$$\begin{array}{r}
 \underline{10} \\
 25 \overline{) 250} \\
 \underline{25} \phantom{0} \\
 00 \\
 \underline{00} \\
 0
 \end{array}$$

Notice that the number to the right of the division sign (÷) goes outside the division block.

When "250 ÷ 25" is changed to 25/250, it is read as "25 divided into 250." Remember, when dividing, you divide into the dividend going from left to right (go from large place value to smaller place value), a change from addition, subtraction, and multiplication.

Set up this problem:

You have 156 eggs. You want to put them into egg cartons (12 eggs per carton). How many cartons do you need?

Solution to  
Frame 1-19

$$\begin{array}{r}
 23042 \\
 \times 1020 \\
 \hline
 46084 \\
 \hline
 23042 \phantom{00} \\
 \hline
 23502840
 \end{array}$$

OR

$$\begin{array}{r}
 23\ 042 \\
 \times 1\ 020 \\
 \hline
 00\ 000 \\
 460\ 840 \\
 0\ 000\ 000 \\
 \hline
 23\ 042\ 000 \\
 23\ 502\ 840 \\
 \hline
 \end{array}$$

(23,502,840)

---

**FRAME 1-21.**

Set up and work this  
problem:  $406 \div 15$

Solution to  
Frame 1-20

$$\begin{array}{r} \underline{\phantom{00}} \\ 12 \overline{) 156} \end{array}$$

---

**FRAME 1-22.**

Let's review the problem given in Frame 1-21 in case you had any problems. Remember, begin at the first (left) digit of the dividend and work to the right.

$15 \overline{) 406}$  How many 15's in 4? Answer: 0  
Write "0" directly above the "4."

$\begin{array}{r} \phantom{0} \\ \underline{0} \\ 15 \overline{) 406} \\ \underline{-0} \\ 4 \end{array}$  Multiply your answer by the divisor and subtract ( $0 \times 15 = 0$ ;  $4 - 0 = 4$ ).

$\begin{array}{r} \phantom{0} \\ \underline{02} \\ 15 \overline{) 406} \\ \underline{-0} \\ 40 \\ \underline{30} \\ 10 \end{array}$  Bring down the next digit (0) from the dividend and put it after the remainder (4). The "working dividend" is now 40.  
How many 15's in 40? Answer: 2. Write the "2" in the quotient immediately above the number previously brought down (the "0"). Multiply your answer by the divisor and subtract ( $2 \times 15 = 30$ ;  $40 - 30 = 10$ ).

What is your next step? \_\_\_\_\_

Solution to  
Frame 1-21

$$\begin{array}{r} \phantom{00} \\ \underline{27r1} \\ 15 \overline{) 406} \end{array}$$

27 with a remainder of one  
OR  
 $27 \frac{1}{15}$   
(twenty-seven and one-fifteenth)

**FRAME 1-23.**

$$\begin{array}{r} \underline{027} \\ 15 \overline{)406} \\ \underline{0} \\ 40 \\ \underline{30} \\ 106 \\ \underline{105} \\ 1 \end{array}$$

After you bring down the "6" from the dividend and enter it after the remainder, you have a new "working dividend" of 106.

How many 15's are in 106? Answer: 7.

Write the 7 over the 6.

Then multiply the divisor by the last answer ( $15 \times 7 = 105$ ) and subtract ( $106 - 105 = 1$ ).

There are no more unused digits in the dividend. If there were, you would continue the above procedures until all of the digits of the dividend have been used.

**NOTE:** Often the initial zero (or zeros) in the quotient is not written. This example shows the zero in the quotient to help make the process clearer. Also, when the numbers contain commas, you may prefer to delete the commas when setting up the division problem.

The answer to how many 15's are in 406 is 27, with 1 left over. The remainder may be written as 27r1. This means "27 with a remainder of 1." More often, the remainder is written as a fraction. The top number of the fraction is the remainder and the bottom number is the divisor (see Frame 1-7). Therefore, the answer to  $406 \div 15$  can be written as

\_\_\_\_\_.

---

Solution to  
Frame 1-22

Bring down the  
next digit from  
the dividend  
(the "6").

**FRAME 1-24.**

If you wanted to express the answer in decimal form rather than using a fraction, put a decimal point after the last digit in the dividend (after the "6"), add zeros to the right of the decimal point (they do not change the value of the dividend), and keep dividing.

**NOTE:** Don't forget to put a decimal point in the quotient directly above the decimal point in the dividend.

Solve the problem so that the quotient is in decimal form instead of "remainder" or "fraction" form. The decimals and the first zero have already been placed. This procedure is discussed in more detail in Lesson 3 (Decimals).

$$\begin{array}{r} \underline{27.} \\ 15 \overline{)406.0} \\ \underline{30} \\ 106 \\ \underline{105} \\ 10 \end{array}$$

---

Solution to  
Frame 1-23

27  $\frac{1}{15}$

**FRAME 1-25.**

Set up and solve this problem:  $2,461 \div 23$ .

Solution to  
Frame 1-24

You haven't finished the problem yet! The answer is  $27.066666666666666$  with the 6's continuing forever. Knowing how to stop (rounding) is covered in Lesson 3.

---

**FRAME 1-26.**

**CHECKING.** Once you have worked a problem, how do you know if you arrived at the correct answer? One way is to do the problem over again from the beginning (and hope that you don't make the same mistake twice). Another way is to check (verify) your answer by rearranging the problem and solving the new problem.

For example, when you subtract, you can check your answer (the difference) by adding the difference to the subtrahend. If your subtraction was correct, the resulting sum will be the minuend.

Look at this problem:

300 (minuend)	Check:	135 (difference)
<u>-165</u> (subtrahend)		<u>+ 165</u> (subtrahend)
135 (difference)		300 (minuend)

Solve and check this problem:

455	Check:
<u>- 50</u>	

Solution to  
Frame 1-25

$$\begin{array}{r} \underline{107} \\ 23 \overline{) 2461} \\ \underline{23} \phantom{00} \\ 16 \phantom{00} \\ \underline{0} \phantom{00} \\ 161 \phantom{00} \\ \underline{161} \phantom{00} \\ 0 \end{array}$$

**FRAME 1-27.**

To check an addition problem, simply reverse the order of the addends and add again. For example:

$$\begin{array}{r} 473 \\ + 361 \\ \hline 834 \end{array} \text{ is checked in this manner: } \begin{array}{r} 361 \\ + 473 \\ \hline 834 \end{array}$$

NOTE: When more than two numbers are added together, there is more than one way to rearrange the addends. The easiest may be to begin with the bottom and add upward (with the top number being added last). The term "adding up" comes from an older method in which the sum was written at the top instead of the bottom.

NOTE: Filling in spaces with zeros can help to keep columns straight.

Solve and check:

$$34 + 121 + 87.$$


---

Solution to  
Frame 1-26

$$\begin{array}{r} 455 \\ - 50 \\ \hline 405 \end{array} \text{ Check: } \begin{array}{r} 405 \\ + 50 \\ \hline 455 \end{array}$$

**FRAME 1-28.**

Just as subtraction problems can be checked using addition, so addition problems can be checked using subtraction. Subtracting one addend from the sum will yield the remaining addend. (This method of checking is used less than the reverse adding method.)

$$\text{Check: } 473 + 361 = 834$$


---

Solution to  
Frame 1-27

$$\begin{array}{r} 11 \\ 34 \\ 121 \\ + 87 \\ \hline 242 \end{array} \text{ Check: } \begin{array}{r} 242 \\ - 121 \\ \hline 034 \\ 11 \end{array}$$

**FRAME 1-29.**

Division can be checked by multiplying the quotient (without the remainder) by the divisor and adding the remainder (if any). The resulting number will be the dividend. For example:

$$\begin{array}{r} 5 \\ 3 \overline{) 17} \\ \underline{15} \\ 2 \end{array} \text{ Check: } \begin{array}{r} 5 \text{ quotient} \\ \underline{\times 3} \text{ divisor} \\ 15 \\ + 2 \text{ remainder} \\ \hline 17 \text{ dividend} \end{array}$$

Solve and check this problem:  $4,864 \div 13$ .

---

Solution to  
Frame 1-28

$$\begin{array}{r} 713 \\ 834 \\ - 473 \\ \hline 361 \end{array} \text{ or } \begin{array}{r} 713 \\ 834 \\ - 361 \\ \hline 473 \end{array}$$

**FRAME 1-30.**

You can also check a division problem by multiplying the divisor by the quotient (without the remainder) and adding the remainder.

Check the division problem in Frame 1-25 using this method.

**NOTE:** If the remainder is expressed as a fraction, multiply the entire quotient by the divisor to obtain the dividend. Multiplying fractions is discussed later.

Solution to  
Frame 1-29

<u>374</u> r2	CHECK
13/ 4864	374
<u>39</u>	<u>x13</u>
96	1122
<u>91</u>	<u>3740</u>
54	4862
<u>52</u>	<u>+2</u>
2	4864

**FRAME 1-31.**

Multiplication can be checked by switching the factors (for example, the product of 232 x 176 should be the same as the product of 176 x 232). Work both problems and see for yourself.

Solution to  
Frame 1-30

23
<u>107</u>
161
<u>23</u>
2461
<u>+0</u>
2461

**FRAME 1-32.**

Multiplication can also be checked by dividing the product by either the multiplicand or the multiplier. If you divide the product by the multiplicand, the quotient will be the multiplier. If you divide the product by the multiplier, the quotient will be the multiplicand.

For example:  $2 \times 3 = 6$  can be checked by dividing:

$$\frac{3}{2/6} \quad \text{OR} \quad \frac{2}{3/6}$$

**Remember** that you may use either one of the factors as the divisor, but the quotient must be the other factor.

Solution to  
Frame 1-31

232	176
<u>176</u>	<u>232</u>
1392	352
16240	5280
<u>23200</u>	<u>35200</u>
40832	40832

Solve and check this problem:  $25 \times 12$

**FRAME 1-33.**

**COMBINED OPERATIONS.** Sometimes a problem requires you to do two or more different operations. For example:  $3 \times 5 + 1$  requires multiplication and addition. Is the answer 16 ( $15 + 1$ ) or 18 ( $3 \times 6$ )? The rule is that, unless the problem indicates otherwise, you should multiply and divide first. After these operations are completed, then you add and subtract.

Practice by solving these problems.

a.  $21 + 5 \times 2 =$  \_\_\_\_\_

b.  $20 \div 5 - 1 =$  \_\_\_\_\_

---

**FRAME 1-34.**

The previous frame tells you to multiply/divide, then add/subtract "unless the problem indicates otherwise." What does this mean?

Sometimes, you need to add or subtract first, then multiply or divide. This is usually indicated by inclosing the operation to be done first in parentheses ( ). For example,  $3 \times 5 + 1 = 16$  because you follow the basic rule of multiply and divide first, but  $3 \times (5 + 1)$  indicates that you are to perform the addition function first. In this instance, the answer is 18 ( $3 \times 6 = 18$ ).

Practice by solving these problems.

a.  $(21 + 5) \times 2 =$  \_\_\_\_\_

b.  $20 \div (5 - 1) =$  \_\_\_\_\_

---

Solution to  
Frame 1-32

25	CHECK
<u>12</u>	<u>12</u>
50	25/ 300
<u>25</u>	<u>25</u>
300	50
	<u>50</u>
	0
OR	
	<u>25</u>
	12/ 300

Solution to  
Frame 1-33

a. 31 ( $21 + 10$ )

b. 3 ( $4 - 1$ )

**FRAME 1-35.**

Sometimes parentheses are used to indicate multiplication. For example,  $6 \times 7$  can also be written as  $(6)(7)$ . Parentheses are often used in algebra in which letters are used to represent numbers. This allows a general formula to be developed. For example, the quantity  $a+b$  multiplied by the quantity  $c+d$  can be represented by:  
 $(a + b) (c + d) = ac + ad + bc + bd$

**NOTE:** "ac" means the quantity "a" multiplied by the quantity "c;" "ad" means the quantity "a" multiplied by the quantity "d;" "bc" means the quantity "b" multiplied by the quantity "c;" and "cd" means the quantity "c" multiplied by the quantity "d." Similarly, "a" times the quantity "b+c" can be written as "a(b+c)."

Test the general formula " $(a + b) (c + d) = ac + ad + bc + bd$ " by letting  $a = 10$ ,  $b = 2$ ,  $c = 30$  and  $d = 4$ ; then check by multiplying  $12 \times 34$ .

---

Solution to  
Frame 1-34

a. 52     $(26 \times 2)$

b. 5     $(20 \div 4)$

***Turn Page for Self-Test***

**FRAME 1-36.**

**SELF TEST.** You have completed the section on adding, subtracting, multiplying, and dividing whole numbers and checking your answers.

If you feel that you need more review on solving and/or checking problems, look over the appropriate frames again. Then work the following self-test exercises shown below. The solutions are found on the following page. **NOTE:** When there is more than one method of checking an answer, only one or two methods may be shown.

1. Set up and solve the problems below.

a.  $455 \times 33 = \underline{\hspace{2cm}}$

b.  $3,690 - 2,460 = \underline{\hspace{2cm}}$

c.  $44 + 275 + 9 = \underline{\hspace{2cm}}$

d.  $400 \div 50 = \underline{\hspace{2cm}}$

2. Solve and check each of the problems below.

a. 
$$\begin{array}{r} \underline{\hspace{1cm}} \\ 3 \overline{)406} \end{array}$$
 Check:

b. 
$$\begin{array}{r} 389 \\ 27 \\ + 122 \\ \hline \end{array}$$
 Check:

c. 
$$\begin{array}{r} 47 \\ \times 22 \\ \hline \end{array}$$
 Check:

d. 
$$\begin{array}{r} 996 \\ - 57 \\ \hline \end{array}$$
 Check:

3. a.  $16 + 4 \div 2 = \underline{\hspace{2cm}}$   
b.  $2(4 + 3) = \underline{\hspace{2cm}}$   
c.  $a(c + d) = \underline{\hspace{2cm}}$

Solution to  
Frame 1-35

408

$$ac + ad + bc + bd =$$

$$(10)(30) + (10)(4) + (2)(30) + (2)(4) =$$

$$300 + 40 + 60 + 8 = 408$$

$$\begin{array}{r} 12 \\ \times 34 \\ \hline 48 \\ 360 \\ \hline 408 \end{array}$$

**Check Your Answers on Next Page**

**SOLUTIONS TO FRAME 1-36 (SELF-TEST)**

1. a. 
$$\begin{array}{r} 455 \\ \times 33 \\ \hline 1365 \\ 13650 \\ \hline 15015 \end{array}$$

b. 
$$\begin{array}{r} 3,690 \\ - 2,460 \\ \hline 1,230 \end{array}$$

c. 
$$\begin{array}{r} 44 \\ 275 \\ \hline 9 \\ \hline 328 \end{array}$$

d. 
$$\begin{array}{r} 50 \overline{) 400} \\ \underline{400} \\ 0 \end{array}$$

2. a. 
$$\begin{array}{r} 135\text{r}1 \\ 3 \overline{) 406} \\ \underline{3} \phantom{0} \\ 10 \\ \underline{9} \phantom{0} \\ 16 \\ \underline{15} \\ 1 \end{array}$$

Check: 
$$\begin{array}{r} 135 \\ \times 3 \\ \hline 405 \\ + 1 \\ \hline 406 \end{array}$$

b. 
$$\begin{array}{r} 389 \\ 27 \\ + 122 \\ \hline 538 \end{array}$$

Check: 
$$\begin{array}{r} 122 \\ 27 \\ + 389 \\ \hline 538 \end{array}$$

c. 
$$\begin{array}{r} 47 \\ \times 22 \\ \hline 94 \\ 94 \\ \hline 1034 \end{array}$$

Check: 
$$\begin{array}{r} 22 \\ \times 47 \\ \hline 154 \\ 88 \\ \hline 1034 \end{array}$$
 OR 
$$\begin{array}{r} 22 \overline{) 1034} \\ \underline{47} \\ 88 \\ \underline{154} \\ 0 \end{array}$$

d. 
$$\begin{array}{r} 996 \\ - 57 \\ \hline 939 \end{array}$$

Check: 
$$\begin{array}{r} 939 \\ + 57 \\ \hline 996 \end{array}$$

3. a.  $18 [6 + 4 \div 2 = 16 + 2 = 18]$

b.  $14 [2 (4 + 3) = 2 \times 7 = 14]$

c.  $ac + ad$  [same as  $(a+b)(c+d)$  with  $b = 0$ ] [using "b" above:  $(2)(4) + (2)(3) = 8 + 6 = 14]$

This lesson may have appeared too simple for you (and perhaps it was), but it serves as a foundation for the lessons that follow. If you have learned other methods of solving these types of problems, you may use them on tests; however, be sure that they work. If you missed any problem(s), review the appropriate lesson frames and rework the problem(s) before going to the next lesson.

***End of Lesson 1***

## LESSON ASSIGNMENT

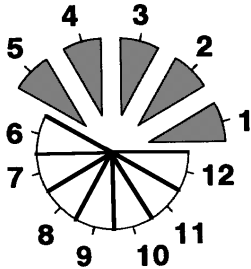
<b>LESSON 2</b>	Fractions.
<b>LESSON ASSIGNMENT</b>	Frames 2-1 through 2-37.
<b>MATERIALS REQUIRED</b>	Pencil, eraser.
<b>LESSON OBJECTIVES</b>	After completing this lesson, you should be able to:  2-1. Write a fraction to describe the part of a whole.  2-2. Add, subtract, multiply, and divide fractions.  2-3. Convert a number to an improper fraction.  2-4. Reduce a fraction.

**SUGGESTION** Work the following exercises (numbered frames) in numerical order. Write the answer in the space provided in the frame. After you have completed a frame, check your answer against solution given in the shaded area of the following frame. The final frame contains review exercises for Lesson 2. These exercises will help you to achieve the lesson objectives.

---

### FRAME 2-1.

**DEFINITION.** A fraction is a part of a whole. If you cut a pie into 12 equal pieces and ate 5 pieces, you would have eaten  $\frac{5}{12}$  (five-twelfths) of the pie ( $\frac{5}{12}$  means "5 parts out of 12 equal parts").



Consider the pie mentioned above. What fraction of the pie still remains?  
(Use the "pie" chart above.)

Remaining pie: \_\_\_\_\_

---

**FRAME 2-2.**

In text, fractions are usually written in a horizontal form, such as "5/12," for ease of reading. When performing calculations, fractions are usually written in a vertical form, such as  $\frac{5}{12}$ .

Solution to  
Frame 2-1.

$$\frac{7}{12}$$

The top (or first) number is called the numerator. The number on bottom (or second number) is called the denominator.

In the fraction 5/12, "5" is the \_\_\_\_\_ and  
"12" is the \_\_\_\_\_.

---

**FRAME 2-3.**

**MULTIPLYING AND DIVIDING FRACTIONS.** It may seem strange to begin with multiplication and division of fractions rather than addition and subtraction. The multiplication and division functions are relatively simple operations, however, and multiplication must be understood before addition and subtraction of some fractions can be performed.

Solution to  
Frame 2-2.

5 numerator  
12 denominator

To multiply fractions:

- (1) Multiply the numerators together;
- (2) Multiply the denominators together; and
- (3) Place the product of the numerators over the product of the denominators.

For example:  $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$

You work this one.  $\frac{1}{3} \times \frac{2}{7} = \frac{1 \times 2}{3 \times 7} = \frac{?}{?}$

---

**FRAME 2-4.**

Let's try four more.

a.  $\frac{1}{2} \times \frac{1}{2} = \underline{\hspace{2cm}}$

b.  $\frac{3}{4} \times \frac{1}{2} = \underline{\hspace{2cm}}$

c.  $\frac{2}{7} \times \frac{1}{8} \times \frac{7}{10} = \underline{\hspace{2cm}}$

d.  $\frac{3}{2} \times \frac{2}{6} = \underline{\hspace{2cm}}$

---

Solution to  
Frame 2-3.

$$\frac{2}{21}$$

**FRAME 2-5.**

When you divide a fraction by another fraction, invert (flip) the second fraction and multiply the fractions.

For example:  $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12}$

To solve the problem  $1/2 \div 1/4$  (one-half divided by one-fourth), you would invert the (choose one --  $1/2$ ;  $1/4$ ) and multiply.

NOTE: Just like  $18 \div 6$  asks, "How many groups of 6 are in 18?", so  $1/2 \div 1/4$  asks, "How many one-fourths are in one-half?" (or, "How many quarters are in a half?").

---

Solution to  
Frame 2-4.

a.  $\frac{1}{4}$       b.  $\frac{3}{8}$

c.  $\frac{14}{560}$     d.  $\frac{6}{12}$

**FRAME 2-6.**

Solve these problems.

a.  $1/4 \div 1/2 =$

b.  $1/2 \div 1/6 =$

c.  $2/3 \div 3/7 =$

d.  $3/7 \div 2/3 =$

---

Solution to  
Frame 2-5.

$\frac{1}{4}$

$1/2 \div 1/4 =$

$1/2 \times 4/1 = 4/2 = 2$

(There are two quarters in a half)

**FRAME 2-7.**

**ADDING AND SUBTRACTING FRACTIONS.** Have you ever heard someone say, "You can't add apples and oranges"? Well, there is a similar rule when working with fractions -- you can't add fractions with different denominators.

This means that, in order for two fractions to be added together, the \_\_\_\_\_ numbers must be the same.

- a. Top
  - b. Bottom
- 

Solution to  
Frame 2-6.

a.  $\frac{2}{4}$       b.  $\frac{6}{2}$

c.  $\frac{14}{9}$       d.  $\frac{9}{14}$

**FRAME 2-8.**

If two fractions have the same denominator (called a "common denominator"), you add the fractions by simply adding the numerators together and putting the sum over the common denominator. YOU DO NOT ADD THE DENOMINATORS TOGETHER. For example:

$$\frac{3}{8} + \frac{1}{8} = \frac{3+1}{8} = \frac{4}{8}$$

More than two fractions can be added together at one time as long as they all have the same denominator. Complete the following exercise.

$$\frac{2}{17} + \frac{1}{17} + \frac{5}{17} + \frac{4}{17} =$$


---

Solution to  
Frame 2-7.

b. Bottom

**FRAME 2-9.**

When you add two fractions with the same denominator together, you add the numerators and put the sum over the common denominator. Likewise, when you subtract two fractions with the same denominator, you subtract the numerators and put the difference over the common denominator.

For example:  $\frac{4}{8} - \frac{1}{8} = \frac{4-1}{8} = \frac{3}{8}$

Solve the following subtraction problems:

a.  $\frac{9}{12} - \frac{5}{12} =$

b.  $\frac{17}{31} - \frac{5}{31} =$

c.  $\frac{5}{10} - \frac{2}{10} =$

---

Solution to  
Frame 2-8.

$$\frac{2+1+5+4}{17} = \frac{12}{17}$$

**FRAME 2-10.**

Adding fractions with common denominators is like adding apples and apples. For example, saying  $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

is much like saying "3 apples plus 1 apple equals 4 apples," with "apples" being "eighths."

But what if you have  $\frac{3}{8} + \frac{1}{4}$  ? What is "3 apples plus 1 orange"?

This problem cannot be solved as long as the fractions are in their present form because they do not have the same \_\_\_\_\_.

---

Solution to  
Frame 2-9.

a.  $\frac{9-5}{12} = \frac{4}{12}$

b.  $\frac{17-5}{31} = \frac{12}{31}$

c.  $\frac{5-2}{10} = \frac{3}{10}$

**FRAME 2-11.**

Have you ever had a friend named James? Some people may call him "James," some may call him "Jim," some may call him "Jimmy," and his little sister may even call him "Bo," but he is the same person regardless of what you call him. Fractions also have many different "names" or forms, and you can change the fraction's name when you need to.

If you can't work with the denominator of a fraction, change the "name" of the fraction until it has the \_\_\_\_\_ that you do want.

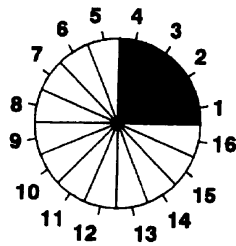
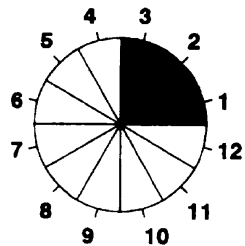
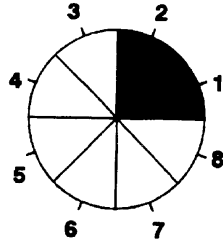
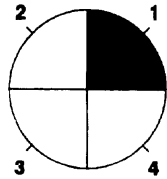
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Solution to  
Frame 2-10.

denominator

**FRAME 2-12.**

The four "pies" shown below are the same size, but have been sliced differently. The same amount of pie has been removed (the shaded area), but the number of slices removed are different. The amount of the shaded area is the same in all four cases, but the name of the fraction is different in each case. Name the shaded areas.



$$\frac{1}{?} = \frac{2}{?} = \frac{?}{12} = \frac{?}{?}$$

Solution to  
Frame 2-11.

denominator

**FRAME 2-13.**

So, if you're "adding apples and oranges," see if you can change the "apple" name of the fraction to the "orange" name (or vice versa).

Consider this problem again:  $\frac{3}{8} + \frac{1}{4}$

Can you change the name of the second fraction (the fraction with the smaller denominator) so that it will have the same denominator as the other fraction? (Refer back to Frame 2-12.)

$$\frac{1}{4} = \frac{?}{8}$$

Solution to  
Frame 2-12.

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$$

**FRAME 2-14.**

If two fractions have different denominators, see if the larger denominator is a multiple of the smaller. That is, can the smaller denominator be multiplied by a whole number and the product be the larger denominator? If so, then the larger denominator can be the common denominator.

8 is a multiple of 4 because  $4 \times \underline{\quad} = 8$ .

---

Solution to  
Frame 2-13.

$$\frac{1}{4} = \frac{2}{8}$$

**FRAME 2-15.**

Therefore, :  $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{?}{8}$

[Adding 2/8 is the same as adding 1/4 since both are forms of the same number (value).]

---

Solution to  
Frame 2-14.

$$4 \times 2 = 8$$

**FRAME 2-16.**

You know that if you multiply a number by "1," then the product is the original number. For example,  $4 \times 1 = 4$ . The same is true if the number is a fraction. For example:  $\frac{1}{2} \times 1 = \frac{1}{2}$

The number "1" has several forms, or names. Some are  $\frac{1}{1}$ ,  $\frac{2}{2}$ ,  $\frac{3}{3}$ , and  $\frac{4}{4}$ .

In each case, the numerator and the denominator are the same. Multiplying a fraction by one of the forms of "1" allows you to change the appearance of the fraction so that it has a different denominator. Now let's find some different names (forms) of the fraction 1/2. (Remember, multiplying a fraction by "1" [regardless of the form of "1" you use] yields a fraction whose actual value has not changed, even if its form has changed.)

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4} ; \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} ; \frac{1}{2} \times \frac{10}{10} = \frac{10}{20} ; \frac{1}{2} \times \frac{3473}{3473} = ?$$


---

Solution to  
Frame 2-15.

$$\frac{5}{8}$$

**FRAME 2-17.**

Let's look at the problem  $\frac{1}{2} + \frac{1}{6}$

The denominators are not the same, so you must find a common denominator. Since "6" is a multiple of "2" ( $2 \times 3 = 6$ ), you can change  $\frac{1}{2}$  to a form that has the same denominator as the other fraction.

$$\frac{1}{2} \times \frac{3}{3} = \frac{?}{?}$$

Solution to  
Frame 2-16.

$$\frac{3+1}{6}$$

**FRAME 2-18.**

Since  $\frac{3}{6}$  is the same as  $\frac{1}{2}$ , you can substitute (switch)  $\frac{3}{6}$  for  $\frac{1}{2}$  and work the problem.

$$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{3+1}{6} = \frac{?}{6}$$

Solution to  
Frame 2-17.

$$\frac{3}{6}$$

**FRAME 2-19.**

Subtraction of fractions works very similar to addition. Find the common denominator, change one or both of the fractions until they have the same denominator, and subtract the numerators.

Work these problems on your own:

$$\frac{1}{2} - \frac{1}{6} =$$

$$\frac{1}{2} - \frac{1}{10} =$$

Solution to  
Frame 2-18.

$$\frac{4}{6}$$

**FRAME 2-20.**

What happens, though, when one denominator is not a multiple of the other. If you can't change the apples to oranges or oranges to apples, maybe you can change them both to grapefruit. That is, find a common denominator to which both denominators can be changed.

The common denominator will be a multiple of the \_\_\_\_\_ of the first fraction and a multiple of the \_\_\_\_\_ of the second fraction.

Solution to  
Frame 2-19.

$$\frac{3-1}{6} = \frac{2}{6}$$

$$\frac{5-1}{10} = \frac{4}{10}$$

**FRAME 2-21.**

Suppose you had two fractions, one with a denominator of "3" and the other with a denominator of "4." One method of getting a common denominator is to multiply the denominators together. For a problem with two denominators ("3" and "4"), a common denominator would be "12" ( $3 \times 4 = 12$  and  $4 \times 3 = 12$ ).

Finish solving the following problem:

$$\frac{1}{3} + \frac{1}{4} = \frac{1}{3} \times \frac{4}{4} + \frac{1}{4} \times \frac{3}{3} = \frac{4}{12} + \frac{?}{12} = ?$$


---

Solution to  
Frame 2-20.

denominator

denominator

**FRAME 2-22.**

Solve this subtraction problem:

$$\frac{5}{6} - \frac{3}{8} =$$


---

Solution to  
Frame 2-21.

$$= \frac{4}{12} + \frac{3}{12} = \frac{4+3}{12} = \frac{7}{12}$$

**FRAME 2-23.**

Although multiplying the denominators together will always give you a common denominator, sometimes a smaller common denominator can be found. Consider the previous problem? Can you think of a common denominator for  $\frac{5}{6}$  and  $\frac{3}{8}$  that is smaller than 48? What number will both 6 and 8 divide into and the quotients be whole numbers (no remainders)?

\_\_\_\_\_

Work the problem  $\frac{5}{6} - \frac{3}{8}$  again using the smaller common denominator. (Divide the denominator into the common denominator and multiply the numerator by the quotient.)

---

Solution to  
Frame 2-22.

$$= \frac{5 \times 8}{6 \times 8} - \frac{3 \times 6}{8 \times 6} = \frac{40-18}{48}$$

$$\frac{22}{48}$$

**FRAME 2-24.**

You can add and subtract fractions in the same problem. Just make sure that each denominator divides evenly into the common denominator. Try this problem.

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$$


---

Solution to  
Frame 2-23.

24

$$= \frac{5 \times 4}{6 \times 4} - \frac{3 \times 3}{8 \times 3} = \frac{20-9}{24} = \frac{11}{24}$$

**FRAME 2-25.**

The solution to Frame 2-24 is shown below (not sufficient space in column to right). The problem is worked two ways. The first shows the problem worked with the common denominator being the product of all of the denominators ( $2 \times 3 \times 5 \times 7 \times 9 = 1890$ ). The second shows the problem being worked with a lower common denominator ( $2 \times 5 \times 7 \times 9 = 630$ ). Did you notice that the denominator "3" divides evenly into the denominator "9"?

Solution to  
Frame 2-24.

$$\frac{633}{1890} \quad \text{or} \quad \frac{211}{630}$$

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} =$$

$$\frac{1 \times 3 \times 5 \times 7 \times 9}{2 \times 3 \times 5 \times 7 \times 9} - \frac{1 \times 2 \times 5 \times 7 \times 9}{3 \times 2 \times 5 \times 7 \times 9} + \frac{1 \times 2 \times 3 \times 7 \times 9}{5 \times 2 \times 3 \times 7 \times 9} - \frac{1 \times 2 \times 3 \times 5 \times 9}{7 \times 2 \times 3 \times 5 \times 9} + \frac{1 \times 2 \times 3 \times 5 \times 7}{9 \times 2 \times 3 \times 5 \times 7} =$$

$$\frac{945}{1890} - \frac{630}{1890} + \frac{378}{1890} - \frac{270}{1890} + \frac{210}{1890} = \frac{(945-630)+(378-270)+210}{1890} =$$

$$\frac{315+108+210}{1890} = \frac{633}{1890}$$

NOTE: There are several ways of adding and subtracting the numerators. One way is shown above. Another (and usually better) way is to add all of the pluses (positives) together, add all of the minuses (negatives) together, and subtract as shown below.

$$(945+378+210) - (630+270) = 1533 - 900 = 633$$

Using 630 ( $2 \times 5 \times 7 \times 9$ ) as the common denominator

$$\frac{1 \times 5 \times 7 \times 9}{2 \times 5 \times 7 \times 9} - \frac{1 \times 2 \times 5 \times 7 \times 3}{3 \times 2 \times 5 \times 7 \times 3} + \frac{1 \times 2 \times 7 \times 9}{5 \times 2 \times 7 \times 9} - \frac{1 \times 2 \times 5 \times 9}{7 \times 2 \times 5 \times 9} + \frac{1 \times 2 \times 5 \times 7}{9 \times 2 \times 5 \times 7} =$$

$$\frac{315}{630} - \frac{210}{630} + \frac{126}{630} - \frac{90}{630} + \frac{70}{630} = \frac{(315-210)+(126-90)+70}{630} =$$

$$\frac{105+36+70}{630} = \frac{211}{630}$$

The converted fractions can also be added and subtracted as below:

$$\frac{(315+126+70) - (210+90)}{630} = \frac{511 - 300}{630} = \frac{211}{630}$$

**FRAME 2-26.**

**REDUCING FRACTIONS.** Usually, you will want your answers "reduced." That is, you will want to use the name (form) of the fraction that has the smallest denominator possible that will still allow both the numerator and denominator to remain whole numbers. The fraction is then "reduced to its lowest form."

Below are three fractions, each in different forms. Circle the reduced form of each fraction.

$$\frac{3}{6} = \frac{1}{2} = \frac{4}{8} = \frac{1/2}{1} ; \quad \frac{2}{3} = \frac{1}{1.5} = \frac{54}{81} ; \quad \frac{2.5}{4} = \frac{10}{16} = \frac{5}{8}$$

---

Solution to  
Frame 2-25.

No problem was given in  
Frame 2-25

**FRAME 2-27.**

To reduce a fraction, find the largest whole number that can be divided into both the numerator and denominator evenly (no remainders). Then divide both the numerator and denominator by that number.

When reducing a fraction you must divide both the \_\_\_\_\_ and the \_\_\_\_\_ by the same number.

Solution to  
Frame 2-26.

$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{5}{8}$$

Remember, both the numerator and the denominator must be whole numbers (not fractions or decimals).

---

**FRAME 2-28.**

For example, the fraction 8/12 can be reduced as shown below.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

Sometimes you may have to divide more than once to reach the reduced form. For instance, the example can also be worked as follows:

$$\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

But this number can be reduced further:  $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

Reduce the following fractions.

$$\frac{5}{10} =$$

$$\frac{27}{30} =$$

$$\frac{107}{107} =$$

$$\frac{3}{17} =$$

**FRAME 2-29.**

Solve the following problems. Reduce the answers.

$$\frac{1}{10} + \frac{3}{10} =$$

$$\frac{2}{3} - \frac{1}{6} =$$

$$\frac{3}{8} \times \frac{2}{3} =$$

$$\frac{7}{8} \div \frac{1}{4} =$$

Solution to  
Frame 2-27.

numerator;  
denominator  
OR  
denominator;  
numerator

Solution to  
Frame 2-28.

$$\frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

$$\frac{27 \div 3}{30 \div 3} = \frac{9}{10}$$

$$\frac{107 \div 107}{107 \div 107} = \frac{1}{1} = 1$$

$$\frac{3 \div 1}{17 \div 1} = \frac{3}{17}$$

**FRAME 2-30.**

Instead of looking for the biggest number that divides into both the numerator and denominator evenly (called the "largest common denominator"), you can divide by **prime** numbers. A prime number is a number that cannot be divided by any whole number other than itself and 1 without leaving a remainder. Prime numbers include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on. Begin with "2." If "2" divides into both the numerator and denominator evenly (no remainder), then reduce the fraction by two. Take the new fraction and try to reduce the new numerator and denominator by "2" again. Continue until the fraction can no longer be reduced by 2. Then do the same with the next prime number ("3"). Continue until there is no whole number (other than 1) that will divide into both the numerator and the denominator evenly. For example:

$$\frac{48}{72} = \frac{48 \div 2}{72 \div 2} = \frac{24}{36} ; \frac{24 \div 2}{36 \div 2} = \frac{12}{18} ; \frac{12 \div 2}{18 \div 2} = \frac{6}{9} ; \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

NOTE: On 6/9, 2 will divide evenly into 6 but not into 9. Therefore, you go to the next prime number.

A variation is to break both the numerator and the denominator down into prime factors (prime numbers that yield the original number when multiplied). If the same factor appears in both the numerator and denominator, mark it out. Mark out factors one at a time. [For example, if you have "2" as a factor 3 times in the numerator but only twice in the denominator, you can only mark out two of the "2's" in the numerator.] When you are finished, multiple the remaining factors to obtain the reduced fraction. For example:

$$\frac{48}{72} = \frac{2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 3 \times 3} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times 2 \times 3 \times 3} = \frac{2}{3}$$

Reduce 200/375 by this method.

---

Solution to  
Frame 2-29.

$$\frac{4}{10} = \frac{2}{5}$$

$$\frac{3}{6} = \frac{1}{2}$$

$$\frac{6}{24} = \frac{1}{4}$$

$$\frac{7 \times 4}{8 \times 1} = \frac{28}{8} = \frac{7}{2}$$

**FRAME 2-31.**

**IMPROPER FRACTIONS.** Notice that the answer to the last problem in Frame 2-29 is unusual in that the fraction has a numerator that is larger than the denominator. Such a fraction is called an improper fraction.

An improper fraction is a fraction in which the numerator is equal to or larger than the denominator.

A proper fraction is a fraction in which the numerator is less than the denominator.

A combination of a whole number and a fraction, such as three and one-half ( $3 \frac{1}{2}$ ), is called a mixed number.

$\frac{3}{5}$  is a(n) \_\_\_\_\_ .

$\frac{5}{3}$  is a(n) \_\_\_\_\_ .

$1 \frac{2}{3}$  is a(n) \_\_\_\_\_ .

---

Solution to  
Frame 2-30.

$$\frac{200}{375} = \frac{2 \times 2 \times 2 \times 5 \times 5}{3 \times 5 \times 5 \times 5} =$$

$$\frac{2 \times 2 \times 2 \times \cancel{5} \times \cancel{5}}{3 \times 5 \times \cancel{5} \times \cancel{5}} = \frac{2 \times 2 \times 2}{3 \times 5} =$$

$$\frac{8}{15}$$

**FRAME 2-32.**

When you worked the problem  $\frac{7}{8}$  divided by  $\frac{1}{4}$ , you came up with an improper fraction ( $\frac{7}{2}$ ) as the answer. When your answer is an improper fraction, you will usually change it to its mixed number form (this is also referred to as "reducing"). To change an improper fraction to a mixed number, divide the numerator by the denominator and put the remainder (if any) over the denominator.

Reduce  $\frac{7}{2}$  to a mixed number.

---

Solution to  
Frame 2-31.

$$\frac{3}{5} \text{ proper fraction}$$

$$\frac{5}{3} \text{ improper fraction}$$

$$1 \frac{2}{3} \text{ mixed number}$$

**FRAME 2-33.**

Reduce the following improper fractions. If the remainder is zero, then the improper fraction reduces to a whole number.

$$\frac{10}{3} =$$

$$\frac{100}{14} =$$

$$\frac{20}{4} =$$

---

Solution to  
Frame 2-32.

$$\frac{7}{2} = \frac{2}{7} \frac{3}{1} = 3 \frac{1}{2}$$

**FRAME 2-34.**

In working some problems, it may be more convenient to multiply by a fraction rather than a mixed number. In such cases, you need to know how to change a mixed number into an improper fraction.

One way of thinking about a mixed number is as a whole number plus a fraction. To change a mixed number to an improper fraction

- (1) Change the whole number to an improper fraction with the same denominator as the fraction, then
- (2) Add the two fractions together.

$$\text{Example: } 5 \frac{2}{3} = \frac{5}{1} + \frac{2}{3} = \frac{5 \times 3}{1 \times 3} + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{15 + 2}{3} = \frac{17}{3}$$

Change the following mixed numbers to improper fractions.

$$2 \frac{1}{2} =$$

$$3 \frac{2}{5} =$$

$$14 \frac{7}{23} =$$


---

Solution to  
Frame 2-33.

$$10/3 = 3 \frac{1}{3}$$

$$100/14 = 7 \frac{2}{14} \\ = 7 \frac{1}{7}$$

$$20/4 = 5$$

**FRAME 2-35.**

A shortcut for changing a mixed number to an improper fraction is to:

- (1) Multiply the whole number by the denominator,
- (2) Add the numerator to the product, and
- (3) Put the sum over the denominator.

$$\text{For example: } 5 \frac{2}{3} = \frac{5 \times 3 + 2}{3} = \frac{15 + 2}{3} = \frac{17}{3}$$

Change the following mixed numbers to improper fractions using the shortcut method:

$$2 \frac{1}{3}$$

$$1 \frac{1}{10}$$

$$4 \frac{2}{5}$$


---

Solution to  
Frame 2-34.

$$2 \frac{1}{2} = \frac{5}{2}$$

$$3 \frac{2}{5} = \frac{17}{5}$$

$$14 \frac{7}{23} = \frac{329}{23}$$

**FRAME 2-36.**

To change a whole number to an improper fraction:

- (1) Choose the desired denominator,
- (2) Multiply the whole number by the denominator, and
- (3) Place the product over the denominator.

Solution to  
Frame 2-35.

$$2 \frac{1}{3} = \frac{7}{3}$$

$$1 \frac{1}{10} = \frac{11}{10}$$

$$4 \frac{2}{5} = \frac{22}{5}$$

For example:

$$3 = \text{how many fifths?} \quad 3 = \frac{3 \times 5}{5} = \frac{15}{5}$$

Fill in the following:

$$2 = \frac{\quad}{2}$$

$$7 = \frac{\quad}{5}$$

$$10 = \frac{\quad}{3}$$


---

**FRAME 2-37.**

Remember "(a + b)(c + d) = ac + ad + bc + bd" from Frame 1-35? This general formula can be applied to multiplying mixed numbers. For example, in the problem  $2 \frac{1}{2} \times 3 \frac{1}{3}$ , let a = 2, b = 1/2, c = 3, and d = 1/3.

Solve the problem using the algebraic formula, then solve it using improper fractions. Your answers should be the same.

---

Solution to  
Frame 2-36.

$$2 = \frac{4}{2}$$

$$7 = \frac{35}{5}$$

$$10 = \frac{30}{3}$$

**Turn Page for Self-Test**

**FRAME 2-38**

**SELF-TEST.** Complete the self-test exercises below. After you have worked all the exercises, turn to the solution sheet on the following page and check your work. For each exercise answered incorrectly, reread the appropriate lesson frame(s) and rework the exercise.

1. Add and reduce:

a.  $\frac{1}{3} + \frac{2}{3} =$

b.  $\frac{1}{2} + \frac{1}{8} =$

c.  $\frac{3}{4} + \frac{1}{3} =$

2. Subtract and reduce:

a.  $\frac{12}{7} - \frac{10}{7} =$

b.  $\frac{7}{8} - \frac{1}{2} =$

c.  $\frac{3}{8} - \frac{1}{3} =$

3. Multiply and reduce:

a.  $\frac{7}{2} \times \frac{2}{5} =$

b.  $\frac{2}{3} \times \frac{1}{4} =$

4. Divide and reduce:

a.  $\frac{2}{5} \div \frac{3}{8} =$

b.  $\frac{1}{5} \div \frac{1}{10} =$

5. Change to improper fractions:

a.  $6 \frac{2}{5} =$

b.  $7 = \frac{\quad}{4}$

Solution to  
Frame 2-37.

$$8 \frac{1}{3}$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

$$(2 + 1/2)(3 + 1/3) =$$

$$(2)(3) + (2)(1/3) + (1/2)(3) + (1/2)(1/3) =$$

$$6 + 2/3 + 3/2 + 1/6 =$$

$$6 + 4/6 + 9/6 + 1/6 =$$

$$6 + 14/6 =$$

$$6 + 2 \frac{2}{6} =$$

$$(6+2) + 1/3 =$$

$$8 \frac{1}{3}$$

CHECK:

$$2 \frac{1}{2} \times 3 \frac{1}{3} =$$

$$5/2 \times 10/3 =$$

$$50/6 = 8 \frac{2}{6} = 8 \frac{1}{3}$$

## SOLUTIONS TO FRAME 2-38 (SELF-TEST)

1. Add and reduce:

$$\text{a. } \frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1$$

$$\text{b. } \frac{1}{2} + \frac{1}{8} = \frac{1 \times 4}{2 \times 4} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{4+1}{8} = \frac{5}{8}$$

$$\text{c. } \frac{3}{4} + \frac{1}{3} = \frac{3 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4} = \frac{9}{12} + \frac{4}{12} = \frac{9+4}{12} = \frac{13}{12} = 1 \frac{1}{12}$$

2. Subtract and reduce:

$$\text{a. } \frac{12}{7} - \frac{10}{7} = \frac{12-10}{7} = \frac{2}{7}$$

$$\text{b. } \frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{1 \times 4}{2 \times 4} = \frac{7}{8} - \frac{4}{8} = \frac{7-4}{8} = \frac{3}{8}$$

$$\text{c. } \frac{3}{8} - \frac{1}{3} = \frac{3 \times 3}{8 \times 3} - \frac{1 \times 8}{3 \times 8} = \frac{9}{24} - \frac{8}{24} = \frac{9-8}{24} = \frac{1}{24}$$

3. Multiply and reduce:

$$\text{a. } \frac{7}{2} \times \frac{2}{5} = \frac{7 \times 2}{2 \times 5} = \frac{14}{10} = 1 \frac{4}{10} = 1 \frac{2}{5}$$

$$\text{b. } \frac{2}{3} \times \frac{1}{4} = \frac{2 \times 1}{3 \times 4} = \frac{2}{12} = \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$$

4. Divide and reduce:

$$\text{a. } \frac{2}{5} \div \frac{3}{8} = \frac{2}{5} \times \frac{8}{3} = \frac{2 \times 8}{5 \times 3} = \frac{16}{15} = 1 \frac{1}{15}$$

$$\text{b. } \frac{1}{5} \div \frac{1}{10} = \frac{1}{5} \times \frac{10}{1} = \frac{1 \times 10}{5 \times 1} = \frac{10}{5} = \frac{10 \div 5}{5 \div 5} = \frac{2}{1} = 2$$

5. Change to improper fractions:

$$\text{a. } 6 \frac{2}{5} = \frac{6 \times 5 + 2}{5} = \frac{30 + 2}{5} = \frac{32}{5}$$

$$\text{b. } 7 = \frac{7 \times 4}{1 \times 4} = \frac{28}{4}$$

**End of Lesson 2**

## LESSON ASSIGNMENT

**LESSON 3**

Decimals.

**LESSON ASSIGNMENT**

Frames 3-1 through 3-36.

**MATERIALS REQUIRED**

Pencil, eraser.

**LESSON OBJECTIVES**

After completing this lesson, you should be able to:

- 3-1. Read decimals.
- 3-2. Write the numerical forms of given word decimals.
- 3-3. Change fractions to decimals.
- 3-4. Change decimals to fractions.
- 3-5. Add, subtract, multiply, and divide decimals.
- 3-6. Round decimals.
- 3-7. Change percents to decimals
- 3-8. Change decimals to percents.
- 3-9. Solve problems involving percents.

**SUGGESTION**

Work the following exercises (numbered frames) in numerical order. Write the answer in the space provided in the frame. After you have completed a frame, check your answer against solution given in the shaded area of the following frame. The final frame contains review exercises for Lesson 3. These exercises will help you to achieve the lesson objectives.

---

**FRAME 3-1.**

**DEFINITION.** A decimal is a number that represents a fraction whose denominator is a power of ten. That is, the denominator is 10 or 100 or 1000 or 10,000, etc.

Being a "power of ten" simply means that the denominator is 10 multiplied by itself a certain number of times. The "power" shows how many times 10 is multiplied by itself to obtain the number. The number 1000, for example, is  $10 \times 10 \times 10$ . This shows that 1000 is 10 multiplied by itself three times. 1000 is 10 to the third power (usually written as  $10^3$ ).

- a. What is the denominator of a fraction if the denominator is equal to 10 to the sixth power? \_\_\_\_\_
- b. What is the denominator of a fraction if the denominator is equal to 10 to the first power?
- 

**FRAME 3-2.**

**READING AND WRITING DECIMALS.** Each digit in a decimal has a place value. A decimal point (period or dot) is used to separate the whole number from the decimal numerals (fraction). Like the place values shown in Frame 1-2, each place value has a name. Like whole numbers, the value decreases by one-tenth ( $1/10$ ) each time you move to the right. (Likewise, the place value increases by 10 if you go to the left.) The names of some of the place values are shown below.

Note: If the entire number has a value that is less than one (there are no whole numbers to the left of the decimal), a zero is usually placed in the ones place to make reading easier (it emphasizes the decimal point).

0	.	1	2	3	4	5	6	7
ones								ten-millionths
decimal point								millionths
		tenths						hundred-thousandths
		hundredths						ten-thousandths
								thousandths

NOTE: Commas are not used to the right of the decimal.

What would you call the eighth and ninth places to the right of the decimal?

---

Solution to  
Frame 3-1.

$$\frac{n}{1,000,000}$$

$$\frac{n}{10}$$

(the "n" represents the numerator.)

**FRAME 3-3.**

As indicated in Frames 1-2 and 3-2, place values have names based upon the powers of ten. Sometimes, they are written as  $10^x$  with the "x" being the power of ten (the number of times ten is multiplied by itself). For example, ten to the third power is one thousand ( $10^3 = 10 \times 10 \times 10 = 1000$ ).

This works for whole numbers, but how about decimals? Think about it as relating to fractions. If the denominator is  $10^3$ , for example, then the fraction would be one-tenth ( $1/10$ ) multiplied by itself ten times ( $1/10 \times 1/10 \times 1/10 = 1/1000$ ).

If the power of ten refers to whole numbers (numerators, if you will), then the power number is expressed as a positive number. If the power of ten refers to a decimal (denominator), then the power number is expressed as a negative number. Negative numbers are denoted by a minus sign; numbers with no negative symbol are assumed to be positive.

$$10^3 = 10 \times 10 \times 10 = 1000 \quad (\text{third power; three zeros})$$

$$10^{-3} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000} = 0.001 \quad (\text{negative three; three places to the right of the decimal})$$

GO TO NEXT FRAME

---

Solution to  
Frame 3-2.

hundred-millionths

billionths

**FRAME 3-4.**

If you combine the information in Frames 1-2, 3-2, and 3-3, you might come up with something like this:

6	5	4	3	2	1	0	.	1	2	3	4	5	6
$10^6$	$10^5$	$10^4$	$10^3$	$10^2$	$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$	$10^{-4}$	$10^{-5}$	$10^{-6}$	

Everything falls into place, except for the "ones" value place, which is also referred to as the "units" place.

What do you think the "?" (unknown power of 10) might be?

---

Solution to  
Frame 3-3.

Frame 3-3 had no problem  
to solve.

**FRAME 3-5.**

$10^0$ , or any number to the zero power, is defined as that number divided by itself. Any number divided by itself is equal to 1 ( $10^0 = 10/10 = 1$ ).

As you probably noticed in Frame 3-2, the place to the right of the decimal point always ends in "ths." The decimal is read as though it were a fraction with the numerator followed by the denominator. (The denominator is the place value of the last digit.) For example:

$$0.46 = \frac{46}{100}, \text{ which is read, "forty-six hundredths."}$$

**NOTE:** Don't forget the "ths." It is this sound which notifies you that you are dealing with a decimal instead of a whole number.

- a. Write the number meaning "one hundred twenty-seven thousand."
  
  
  
  
  
  
  
  
  
  
  - b. Write the number meaning "one hundred twenty-seven thousandths."
- 

Solution to  
Frame 3-4.

If you said "zero" ( $10^0$ ),  
then you would be correct.

**FRAME 3-6.**

Remember, the "ths" of the right-most digit is read. For example, 0.032 is read as "thirty-two thousandths," not as "three hundredths and two thousandths."

- a. How is 0.3736 read? \_\_\_\_\_.
  
  
  
  
  
  
  
  
  
  
  - b. How is 0.000002 read? \_\_\_\_\_.
- 

Solution to  
Frame 3-5.

- a. 127,000
  
- b. 0.127 or  $\frac{127}{1000}$

**FRAME 3-7.**

The decimal point is read as "AND." For example, 35.362 is read as "thirty-five AND three hundred sixty-two thousandths."

- a. How is 404.404 read? \_\_\_\_\_
  
  
  
  
  
  
  
  
  
  
  - b. Write fourteen and five tenths. \_\_\_\_\_
- 

Solution to  
Frame 3-6.

- a. three thousand  
seven hundred  
thirty-six  
ten-thousandths
  
- b. 2 millionths

**FRAME 3-8.**

**CHANGING FRACTIONS TO DECIMALS.** Fractions can be changed to a decimal by dividing the numerator by the denominator.

The steps for changing a fraction into a decimal form are:

- (1) Write a division problem in which the numerator is divided by the denominator.
- (2) Place a decimal point to the right of the numerator.
- (3) Add zeros to the right of the decimal point, as needed.
- (4) Place a decimal point in the quotient **DIRECTLY OVER** the decimal point in the division bracket.
- (5) Divide as normal (see Frames 1-20 through 1-24).
- (6) Continue dividing until your remainder is zero or until you have reached the needed level of accuracy. [Some problems, such as  $\frac{2}{3}$ , never have a remainder of zero. You have already run across one such problem in Frame 1-24.]

Change  $\frac{7}{8}$  to a decimal.

---

Solution to  
Frame 3-7.

- a. four hundred four  
and four hundred  
four thousandths

Note: "And" can mean "decimal" or "plus." If you said "four hundred and four and four hundred and four thousandths," you would really be saying  $400 + 4 + 400.004$ .

- b. 14.5

**FRAME 3-9.**

Let's try a couple more problems. Carry out your division though four places to the right of the decimal (no further than the ten-thousandths place).

- a. Change  $\frac{1}{12}$  to a decimal.
- b. Change  $\frac{8}{900}$  to a decimal.

Solution to  
Frame 3-8.

$$\begin{array}{r} \underline{0.875} \\ 8 \overline{) 7.000} \\ \underline{0} \phantom{00} \\ 7 \phantom{0} \\ \underline{6} \phantom{4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

**NOTE:** If a digit in the quotient is zero and it is **TO THE RIGHT OF THE DECIMAL POINT**, the zero must be written in the quotient.

---

**FRAME 3-10.**

If you have a mixed number and wish to convert the fraction to a decimal, then the whole number goes to the left of the decimal and the fraction goes to the right of the decimal. For example "five and one-half" (or "five and five-tenths") is written as "5.5."

- a. Write  $3 \frac{1}{12}$  as a decimal (carry out to the fourth decimal place).
- b. Write  $300 \frac{8}{900}$  as a decimal (carry out to the fourth decimal place).
- 

Solution to  
Frame 3-9.

- a. 0.0833  
b. 0.0088

**FRAME 3-11.**

To change an improper fraction to a decimal, divide the numerator by the denominator. Remember to keep the decimal in the quotient above the decimal point in the dividend. The improper fraction  $\frac{3}{2}$  is shown below being changed to its decimal form.

$$\begin{array}{r} \underline{1.5} \\ 2 \overline{) 3.0} \\ \underline{2} \phantom{0} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Change  $\frac{19}{8}$  to a decimal.

---

Solution to  
Frame 3-10.

- a. 3.0833  
b. 300.0088

**FRAME 3-12.**

**CHANGING DECIMALS TO FRACTIONS.** Frames 3-1 through 3-7 have given you the basic information you need to change a decimal to a fraction. Just put the numerator over the appropriate denominator (a power of 10).

For example:  $0.045 = 45 \text{ thousandths} = \frac{45}{1000}$

If you want the fraction reduced, divide the numerator and denominator by their common factors (whole numbers which divide into both the numerator and denominator without leaving a remainder). For example

$$\frac{45}{1000} = \frac{45 \div 5}{1000 \div 5} = \frac{9}{200}$$

- a. Change 0.004 to a fraction and reduce.
- b. Change 0.0031 to a fraction and reduce.
- 

Solution to  
Frame 3-11.

$$\frac{19}{8} = 2.375$$

**FRAME 3-13.**

**ADDING DECIMALS.** Adding decimals is much the same as the addition of whole numbers. The difference is that there is a decimal point to keep in mind. The decimals are put in a straight column; that is, **DECIMAL POINTS ARE UNDER DECIMAL POINTS** (see example). The decimal point is brought down to the sum, and the addition is carried on just as it is in whole number addition. Rules for carrying still apply.

Example:

$$\begin{array}{r} \phantom{1} \\ 6.9 \\ 0.01 \\ \hline 22.2201 \\ 29.1301 \end{array}$$

**NOTE:** Zeros may be added after the last number of a decimal to help keep the digits in the proper alignment (or columns) as shown

$$\begin{array}{r} \phantom{1} \\ 6.9000 \\ 0.0100 \\ \hline 22.2201 \\ 29.1301 \end{array}$$

Add these decimals:  $33.79 + 0.0097 + 2.4 + 6$

Solution to  
Frame 3-12.

- a.  $4/1000 = 1/250$
- b.  $31/10000$  (will not reduce)

---

**FRAME 3-14.**

**SUBTRACTING DECIMALS.** The rules for subtracting decimals are basically the same as they are in the subtraction of whole numbers. Just as in the addition of decimals, the decimal points must be lined up under each other. If the top number (minuend) has fewer decimal places than the number on the bottom (subtrahend), then you must add zeros after the last digit of the top number until both numbers have the same number of places to the right of the decimal. Rules for "borrowing" still apply.

Solve these problems:

a.  $729.75309 - 0.0077$

b.  $3 - 0.003$

Solution to  
Frame 3-13.

$$\begin{array}{r} \phantom{1} \phantom{1} \\ 33.79 \\ 0.0097 \\ 2.4 \\ \hline 6. \\ \hline 42.1997 \end{array}$$

**FRAME 3-15.**

**MULTIPLYING DECIMALS.** Decimals are multiplied just as whole numbers are, except you have to put a decimal in the final answer (product). Unlike adding and subtracting decimals, however, you do not need to keep the decimals lined up (aligned) nor do you have to add zeroes to make the number of decimal places equal.

Sample problem:  $1.11 \times 0.15$

Multiply just as you do in whole numbers.  
(Ignore the decimal for now.)

$$\begin{array}{r} 1.11 \quad 111 \\ \times 0.15 \quad \times 15 \\ \hline 555 \\ 111 \\ \hline 1665 \end{array}$$

NOTE: Normally, no space is left under the decimal when working a multiplication problem.

Now, you need to place the decimal point in the answer. To do this, you:

- (1) Count the number of digits to the right of the decimal point in the top factor (multiplicand),
- (2) Count the number of digits to the right of the decimal point in the bottom factor (multiplier),
- (3) Add the results together,
- (4) Count off that many places from the **RIGHT** in the **PRODUCT** (number at far right is one, number immediately to its left is two, etc.) , and
- (5) Place a decimal point to the left of that location.

In the example, the multiplicand (1.11) has 2 places to the right of the decimal and the multiplier (0.15) has 2 places to the right of the decimal. Adding the results (2 places + 2 places = 4 places) tells how many decimal places (places to the right of the decimal) you have in the produce. The answer then is \_\_\_\_\_.

---

**FRAME 3-16.**

Place the **DECIMAL POINT** in the product of this problem:

$$\begin{array}{r} 3.217 \\ \times 4.71 \\ \hline 3217 \\ 22519 \\ 12868 \\ \hline 1515207 \end{array}$$

Solution to  
Frame 3-14.

$$\begin{array}{r} \phantom{0} 4 \phantom{0} 12 \phantom{0} 10 \\ \text{a. } 729.75399 \\ - \phantom{0} 0.00770 \\ \hline 729.74539 \end{array}$$

$$\begin{array}{r} \phantom{0} 2 \phantom{0} 9 \phantom{0} 9 \phantom{0} 10 \\ \text{b. } 3.000 \\ + \phantom{0} 0.003 \\ \hline 2.997 \end{array}$$

Solution to  
Frame 3-15.

0.1665 (the answer must have four decimal places to the right of the decimal point)

**FRAME 3-17.**

Work the following problems. Be sure to place the decimal point correctly in the product.

**NOTE:** If there are not enough digits in the product, put zeros to the left of the product until you have enough digits to place the decimal. Normally, you will also put another zero to the left of the decimal to indicate that there are no whole numbers.

$$\begin{array}{r} \text{a. } 0.0035 \\ \times 3.28 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b. } 22.222 \\ \times 0.11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c. } 0.001 \\ \times 0.1 \\ \hline \end{array}$$

Solution to  
Frame 3-16.

15.15207 (the multiplicand has 3 decimal places and the multiplier has 2 decimal places.  
 $3 + 2 = 5$ , so the product must have five decimal places to the right of the decimal point)

**FRAME 3-18.**

**DIVIDING DECIMALS.** A very important rule in dividing by a decimal is that the divisor must be "changed" into a whole number before division is started. This is done by moving the decimal point in the divisor all the way to the right (that is, make the divisor a whole number).

For example:  $0.25 / \underline{\quad}$  becomes  $25. / \underline{\quad}$  by moving the decimal two places to the right. In reality, you have just multiplied the divisor by 100. In order to offset this change, you must multiply the dividend by 100 also. To do this, move the decimal in the dividend the SAME NUMBER of places to the right. Add zeros to the dividend as needed.

For example:  $0.25 / \underline{1.25}$  becomes  $25. / \underline{125}$ .

If you prefer, think of the problem as a fraction in which the denominator must be changed to a whole number.

$$\text{For example: } \frac{1.25}{0.25} \times \frac{100}{100} = \frac{125}{25}$$

Solution to  
Frame 3-17.

$$\begin{array}{r} \text{a. } 0.0035 \text{ (4 places)} \\ \times 3.28 \text{ (2 places)} \\ \hline 0.011480 \text{ (6 places)} \end{array}$$

(Note: When an answer has a zero in the last decimal place, the zero is often dropped -- 0.01148)

$$\begin{array}{r} \text{b. } 22.222 \text{ (3 places)} \\ \times 0.11 \text{ (2 places)} \\ \hline 2.44442 \text{ (5 places)} \end{array}$$

$$\begin{array}{r} \text{c. } 0.001 \text{ (3 places)} \\ \times 0.1 \text{ (1 place)} \\ \hline 0.0001 \text{ (4 places)} \end{array}$$

Rewrite the following problems to remove the decimal in the divisor.

$$3.3 / \underline{0.066}$$

$$0.0033 / \underline{66}$$

$$0.000033 / \underline{0.0066}$$

**FRAME 3-19.**

Complete the problems by dividing. Don't forget to put your decimal point in the quotient directly above the decimal point in the dividend.

Remember: Zeros between the decimal point and the non-zero numbers to the right of the decimal must be written.

a.  $3.3 \overline{) 0.066}$

b.  $0.0033 \overline{) 66}$

c.  $0.000033 \overline{) 0.0066}$

Solution to  
Frame 3-18.

$$33 \overline{) 0.66}$$

(moved decimals one place)

$$33 \overline{) 660000}$$

(moved decimals four places)

$$33 \overline{) 6600}$$

(moved decimals six places)

---

**FRAME 3-20.**

For additional practice, divide "1" by:

a. 0.1

b. 0.001

c. 0.000001

Solution to  
Frame 3-19.

a. 0.02

b. 20,000

c. 200

Remember to put a decimal point after the "1" in the dividend and add zeroes to the right as needed.

---

**FRAME 3-21.**

**"ROUNDING" DECIMALS.** In many cases, a large, cumbersome, accurate decimal value is not necessary. In cases when less accuracy (fewer digits to the right of the decimal point) will do, you may round (or round off) the decimal.

To make a long decimal number shorter and easier to use without losing too much accuracy, you can \_\_\_\_\_ the number.

Solution to  
Frame 3-20.

a.  $1/0.1 = 10$

b.  $1/0.001 = 1000$

c.  $1/0.000001 = 1,000,000$

**FRAME 3-22.**

Suppose jellybeans cost \$1.99 per pound and you bought exactly a quarter-pound (0.25 pounds). How much money should you give the clerk? [Assume that there is no tax on the purchase.]

$$\begin{array}{r} 1.99 \text{ (2 decimal places)} \\ \underline{0.25 \text{ (2 decimal places)}} \\ 9 \text{ 95} \\ \underline{39 \text{ 8}} \\ 49 \text{ 75 (4 decimal places)} \end{array}$$

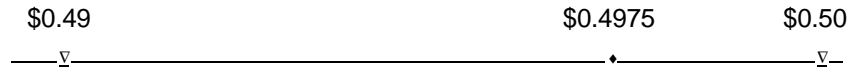
The answer is \$0.4975 . If you had changed \$1.99 into pennies when you started, then you would owe 49.75 (49 3/4) pennies. You could take a penny, divide it into four equal parts, and give the clerk three of them (along with the 49¢), but I don't think the clerk will be very happy. Instead, we usually round the cost to the nearest penny. That means \$0.4975 will be rounded to the nearest \_\_\_\_\_ of a dollar (penny or cent).

Solution to  
Frame 3-21.

round (off)

**FRAME 3-23.**

In the previous example, you will pay either \$0.49 or \$0.50 (the amount just below the calculated true price and the amount just above the calculated true price). Look at the number line drawn below.



Is 0.4975 (•) closer to 0.49 or 0.50? \_\_\_\_\_

Solution to  
Frame 3-22.

hundredth

**FRAME 3-24.**

Therefore, 0.4975 rounded to the nearest hundredth is 0.50. You owe the clerk \$0.50 for the candy.

The previous example showed a problem that required rounding to the nearest hundredth. Other problems may involve rounding to the nearest tenth, to the nearest thousandth, to the nearest millionth, to the nearest whole number, to the nearest thousand, to the nearest billion, etc.

As long as you are dealing with whole numbers and/or decimals, you can use some basic steps to determine how to round a given number. These steps are given in the next frame.

Remember, you will be rounding to the nearest number. The theory is that if several numbers are rounded, some will go to the higher number (round up) while others will go to the lower number (round down). If all of the original (unrounded) numbers were added together and rounded, the results should be about the same as the sum of the rounded numbers.

---

Solution to  
Frame 3-23.

0.50

**FRAME 3-25.**

Rounding to the nearest number or value involves these steps:

- (1) Determine the PLACE you want to round to (tenths, hundredths, etc. -- call it the "place value to be retained)."
- (2) Locate the digit in that place value (call this the "digit to be rounded."
- (3) Either leave that digit unchanged (round down) or increase that digit by 1 (round up) using these rules.
  - (a) Locate the digit directly to the right of the digit to be rounded.
  - (b) If that digit less than 5 (that is, a 0, 1, 2, 3, or 4), leave the digit to be rounded unchanged.
  - (c) If that digit is 5 or more (that is, a 5, 6, 7, 8, or 9), increase the digit to be rounded by one (1).
- (4) Once you have rounded up or down, drop all of the digits to the right of the place value to be retained (the rounded digit).

Solution to  
Frame 3-24.

No problem was given in  
this frame.

For example, round 28.034697 to the nearest thousandth.

- a. What is the digit in the place value to be retained? \_\_\_\_\_
- b. What is the digit to the immediate right of that digit? \_\_\_\_\_
- c. Based upon the information in "b," should the digit in the place value to be retained be left unchanged or be increased by 1? \_\_\_\_\_
- d. What is 26.034697 rounded to the nearest thousandth?  
\_\_\_\_\_

**FRAME 3-26.**

Round 28.034697 to the nearest:

- a. hundredth \_\_\_\_\_
  - b. ten-thousandth \_\_\_\_\_
  - c. hundred-thousandth \_\_\_\_\_
  - d. whole number \_\_\_\_\_
  - e. ten \_\_\_\_\_
  - f. hundred \_\_\_\_\_
- 

Solution to  
Frame 3-25.

- a. 4 (in the thousandths position)
- b. 6 (the ten-thousandths position)
- c. Increased (6 is 5 or more)
- d. 28.035

**FRAME 3-27.**

Did you have any problems? The information given below may help if you did.

- a. Round 28.034697 to the nearest hundredth.

The digit to be rounded is 3. The digit to the right of the hundredths is 4, so you leave the 3 unchanged.

**NOTE:** Even though this digit rounded to 5 when you rounded to the thousandths position in the previous problem, you must use the actual (unrounded) digit when working this problem.

- b. Round 28.034697 to the nearest ten-thousandth.

The digit to be rounded is 6. The digit to the right is 9. Round up to 7.

- c. Round 28.034697 to the nearest hundred-thousandth.

The digit to be rounded is 9. The digit to the right is 7. Round up. When you add 1 to 9, you get 10. Write down the zero and carry the one.  $26.03469 + 0.00001 = 26.03470$ . When writing the answer, you can include the zero at the end (26.03470) or drop the zero (26.0347).

- d. Round 28.034697 to the nearest whole number.

The digit to be rounded is in the units (ones) position, which is 8. The digit to the right is 0 (tenths position). Round down.

- e. Round 28.034697 to the nearest ten.

The digit to be rounded is in the tens position, which is 2. The digit to the right is 8 (units position). Round up. The 2 becomes 3, but you cannot just drop the digits as you do when the number to be rounded is to the right of the decimal. Although the digits are dropped, the place values to the left of the decimal must be shown. They are filled with zeroes. Digits to the right of the decimal are dropped without putting zeroes in their place.

- f. Round 28.034697 to the nearest hundred

The digit to be rounded is in the hundreds position, which has no number now. Change 28.034697 to 028.034697 (adding a zero to the front does not change the value of the number). The number to be rounded is now 0. The digit to the right is 2 (tens position). Round down. The 0 remains zero. Like exercise "e" above, you put zeroes in place of the digits to the left of the decimal that are being dropped. The result is "000," which is usually written as just "0."

Solution to  
Frame 3-26.

a. 28.03

b. 28.0347

c. 28.03470

d. 28

e. 30

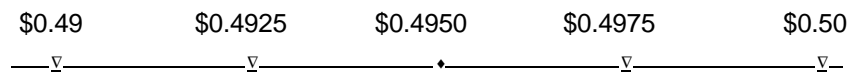
f. 000 (or just 0)

**FRAME 3-28.**

**SPECIAL ROUNDING PROCEDURES.** In the preceding frames, you used rules to round to the nearest place (nearest hundredth, nearest whole number, etc.). The basic theory is that sometimes you round up and sometimes you round down, but in the end it balances out.

Some organizations, however, may use different rules. For example, suppose you are rounding off the weights of individual products to the nearest pound, then adding the weights together to determine the total weight of the shipment. One organization may not care if the estimated weight is more than the actual total weight, but will be very upset if the estimate is below the actual weight. In such a case, you may be told to round up at all times to prevent an underestimate. Likewise, you may be told to always round down by an organization that must make sure that the actual weight is not under your estimate.

Some other organizations may use modified rules of rounding. Refer back to Frame 3-23. Suppose that the amount you wished to round to the nearest cent (\$0.01) had been exactly in the middle (0.\$4950).



According to our rules of rounding, you would always round half-cents up to the next penny. But suppose you knew that you would have a lot of halves (say a lot of half pounds in the above example). You might want a system to round the halves up sometimes and round them down sometimes. One such rule is the "engineer's rule of rounding." When using this modified rule of rounding, if the digit(s) to the right of the digit to be rounded is "5" or "50", you round down if the digit to be rounded is even (0, 2, 4, 6, or 8) and round up if the digit is odd (1, 3, 5, 7, or 9).

For example, 2 1/2 (2.5) pounds rounded to the nearest pound would round to 2 pounds (the 2 is even). 3 1/2 (3.5) pounds, however, would round to 4 pounds (the 3 is odd).

If you used the engineer's rule of rounding given above to round \$0.4950 to the nearest penny, the results would be (chose one -- \$0.49 \$0.50).

**NOTE:** The information presented in this frame was for your information. In this subcourse, you will only be tested on the rounding rules given in Frame 3-25 and not on any system of rounding presented in this frame.

**NOTE:** Do not use any of the rules of rounding presented in this frame unless specifically told to do so.

Solution to  
Frame 3-27.

No problem was given in  
this frame.

**FRAME 3-29.**

**PERCENTS.** A percent (%) is a special type of decimal form. Percent means "per one hundred." It tells how many hundredths. (Think of cents. One cent is  $1/100$  [1%] of a dollar.)

For example, 24 percent means  $24/100$ , which is 0.24.

To change from a percent to a decimal, simply move the decimal point two pieces to the left. If no decimal point is shown, put one after the last digit. Add zeroes to the left of the percentage number if needed. For example,  $2\% = 0.02$ .

Solution to  
Frame 3-28.

\$0.50

The number to be rounded (9) is odd, so you round up (add 1 to the number to be rounded).

Change these percent forms to their decimal forms.

a.  $20\% =$  \_\_\_\_\_

b.  $5.5\% =$  \_\_\_\_\_

c.  $1/8\% =$  \_\_\_\_\_

d.  $0.2\% =$  \_\_\_\_\_

e.  $350\% =$  \_\_\_\_\_

---

**FRAME 3-30.**

In math problems, the word "of" frequently indicates that you are to multiply. Solve these problems by changing the percent to a decimal and multiplying. Round any answer involving money to the nearest cent.

a. 20% of \$300 \_\_\_\_\_

b. 8.5 % of \$255 \_\_\_\_\_

c. 150% of 10 \_\_\_\_\_

d.  $1/2\%$  of 1000 \_\_\_\_\_

Solution to  
Frame 3-29.

a. 0.20 (or 0.2)

b. 0.055

c. 0.00125  
(Change the fraction to a decimal form, then move the decimal point 2 places to the left.)

d. 0.002

e. 3.50 (or 3.5)

**FRAME 3-31.**

To change from a decimal form to a percent form, move the decimal two places to the right and add the percent symbol (%). For example, the percent form of 0.25 is 25%.

Change from the decimal form to the percent form. Add zeroes as needed.

a.  $0.5 =$  \_\_\_\_\_

b.  $0.153 =$  \_\_\_\_\_

c.  $1.25 =$  \_\_\_\_\_

d.  $0.0003 =$  \_\_\_\_\_

---

Solution to  
Frame 3-30.

a. \$60 (or \$60.00)

b. \$21.68

c. 15

d. 5

**FRAME 3-32.**

Sometimes you are asked what percent one number is of another. For example, what percent of 20 is 5? (or 5 is what percent of 20?)

To solve, change the information to a fraction, then to a decimal, then to a percent. For example:

$$5/20 = 0.25 = 25\%$$

Solve these problems. Round to the nearest hundredth of a percent, if needed.

a. What percent of 100 is 3? \_\_\_\_\_

b. 120 is what percent of 60? \_\_\_\_\_

c. 1 is what percent of 3? \_\_\_\_\_

---

Solution to  
Frame 3-31.

a. 50%

b. 15.3%

c. 125%

d. 0.03%

**FRAME 3-33.**

A variation of the above problems is to tell you that a number is a certain percent of the original number, then ask you to find the original number.

For example, 25% of what number is 5? (or 5 is 25% of what number?) Let "N" stand for the original number. The question can then be restated as 25% of N is 5. The mathematical form of this statement is  $25\% \times N = 5$ . You can either state the problem in decimal form or as a fraction:  $0.25N = 5$  or  $1/4 \times N = 5$

NOTE: 0.25N is another way of writing (0.25)(N). 1/4 is the reduced form of 25/100.

State "Twenty percent of what number is 30?" as a decimal or fractional equation.

---

Solution to  
Frame 3-32.

- a. 3% (3/100)
- b. 200% (120/60)
- c. 33.33% (1/3)

**FRAME 3-34.**

After you have stated the problem as an equation (that is, the mathematical statements on both sides of the "=" symbol are equal), solve for N. Multiply or divide both sides of the equation by the same number or fraction in order to change one side of the equation to N (or 1N). The example can be worked as shown.

$$0.25N = 5 \qquad \text{or} \qquad 1/4 \times N = 5$$

$$\frac{0.25N}{0.25} = \frac{5}{0.25} \qquad \frac{N}{4} \times \frac{4}{1} = \frac{5}{1} \times \frac{4}{1}$$

$$1N = 20 \qquad \frac{4N}{4} = \frac{5 \times 4}{1}$$

$$N = 20$$

Solution to  
Frame 3-33.

$$0.20N = 30$$

or

$$1/5 \times N = 30$$

Solve the problem: "Twenty percent of what number is 30?"

---

**FRAME 3-35.**

To check your answer, simply substitute your answer for N in the equation.

For example: 25% of what number is 5?

$$0.25 \times N = 5 \qquad \text{or} \qquad 1/4 \times N = 5$$

$$0.25 \times 20 = 5 \qquad \qquad \qquad 1/4 \times 20 = 5$$

$$5 = 5 \qquad \qquad \qquad 20/4 = 5$$

$$5 = 5$$

Solution to  
Frame 3-34.

$$0.20N = 30$$

$$N = 30/0.20 = 150$$

or

$$1/5 \times N = 30$$

$$N = 30 \times 5 = 150$$

Check your answer to Frame 3-34.

---

***Turn Page for Self-Test***

**FRAME 3-36.**

**SELF TEST.** You have completed the section on adding, subtracting, multiplying, and dividing decimals, changing fractions to decimals, changing decimals to fractions, and working with percents.

If you feel that you need review on any of the above, reread the appropriate frames. Then work the following self-test exercises on this and the following page. The solutions are found on the pages following the exercises.

1. Write the numerical form of the following word decimals:

- a. Nine and seventy-five hundredths \_\_\_\_\_
- b. Twelve and three tenths \_\_\_\_\_
- c. Seventy and three thousandths \_\_\_\_\_
- d. Seventy-three thousandths \_\_\_\_\_

2. Change the fractions below to decimals:

- a.  $\frac{3}{10}$  \_\_\_\_\_
- b.  $\frac{4}{5}$  \_\_\_\_\_
- c.  $\frac{3}{4}$  \_\_\_\_\_
- c.  $\frac{5}{2}$  \_\_\_\_\_

3. Change the decimal forms below to fractions. Reduce the fractions.

- a.  $0.25 =$  \_\_\_\_\_
- b.  $0.105 =$  \_\_\_\_\_
- c.  $0.9 =$  \_\_\_\_\_
- d.  $0.035 =$  \_\_\_\_\_

4. Round the following decimal forms as directed.

Nearest tenth

Nearest thousandth

- a.  $0.6454 =$  \_\_\_\_\_
- b.  $0.7821 =$  \_\_\_\_\_
- c.  $3.15091 =$  \_\_\_\_\_
- d.  $8.7461 =$  \_\_\_\_\_
- e.  $0.9659 =$  \_\_\_\_\_
- f.  $3.9997 =$  \_\_\_\_\_

Solution to  
Frame 3-35.

$$20\% \text{ of } 150. = 30$$

$$0.20 \times 150 = 30$$

$$30 = 30$$

or

$$\frac{1}{5} \times 150 = 30$$

$$150/5 = 30$$

$$30 = 30$$

5. Add the following decimals:

a.  $9.37 + 15.756 + 0.76 =$  \_\_\_\_\_

b.  $69.333 + 0.12 + 111.1 =$  \_\_\_\_\_

c.  $0.0055 + 7.02 + 12.367 =$  \_\_\_\_\_

6. Subtract the following decimals:

a.  $13.14 - 2.96 =$  \_\_\_\_\_

b.  $0.7068 - 0.077 =$  \_\_\_\_\_

c.  $447.3 - 0.93 =$  \_\_\_\_\_

7. Multiply the following decimals:

a.  $0.03 \times 10.31 =$  \_\_\_\_\_

b.  $0.71 \times 0.004 =$  \_\_\_\_\_

c.  $1.51 \times 0.712 =$  \_\_\_\_\_

8. Divide the following decimals:

a.  $0.08 \div 0.004 =$  \_\_\_\_\_

b.  $0.00344 \div 3.44 =$  \_\_\_\_\_

c.  $0.04 \div 0.08 =$  \_\_\_\_\_

9. Change the following to their decimal forms.

a.  $37\% =$  \_\_\_\_\_

b.  $6\% =$  \_\_\_\_\_

c.  $350\% =$  \_\_\_\_\_

d.  $1.25\% =$  \_\_\_\_\_

10. Change the following to their percent forms.

a.  $3.1 =$  \_\_\_\_\_

b.  $0.27 =$  \_\_\_\_\_

c.  $0.003 =$  \_\_\_\_\_

d.  $1/5 =$  \_\_\_\_\_

11. Solve these problems.

a. What is 30% of 50? \_\_\_\_\_

b. 20 is what percent of 25? \_\_\_\_\_

c. 27 is 25% of what number?

***Check Your Answers on Next Page***

### SOLUTIONS TO FRAME 3-36 (SELF-TEST)

1. a. 9.75  
b. 12.3  
c. 70.003  
d. 0.073

2. a. 0.3            b. 0.8  
c. 0.75           d. 2.5

3. a.  $25/100 = 1/4$             b.  $105/1000 = 21/200$   
c.  $9/10$                         d.  $35/1000 = 7/200$

4. a. 0.6                        d. 8.746  
b. 0.8                        e. 0.966  
c. 3.2                         f. 4.000

- |                |                 |                 |
|----------------|-----------------|-----------------|
| 5. a. 09.370   | b. 069.333      | c. 00.0055      |
| 15.756         | 000.120         | 07.0200         |
| <u>+00.760</u> | <u>+111.100</u> | <u>+12.3670</u> |
| 25.886         | 180.553         | 19.3925         |

- |               |                |                |
|---------------|----------------|----------------|
| 6. a. 13.14   | b. 0.7068      | c. 447.30      |
| <u>-02.96</u> | <u>-0.0770</u> | <u>-000.93</u> |
| 10.18         | 0.6298         | 446.37         |

7. a. 0.3093  
b. 0.00284  
c. 1.07512

8. a. 20  
b. 0.001  
c. 0.5

9. a. 0.37  
b. 0.06  
c. 3.50 (or 3.5)  
d. 0.0125
10. a. 310%  
b. 27%  
c. 0.3%  
d. 20% ( $1/5 = 0.20 = 20\%$ )
11. a.  $0.30 \times 50 = \underline{15}$   
b.  $20 = N \times 25$   
 $20/25 = N$   
 $0.8 = N = \underline{80\%}$   
c.  $27 = 25\% \times N$   
 $27 = 0.25N$   
 $27/0.25 = 0.25N/0.25$   
 $\underline{108} = N$

***End of Lesson 3***

## LESSON ASSIGNMENT

<b>LESSON 4</b>	The Metric System.
<b>LESSON ASSIGNMENT</b>	Frames 4-1 through 4-49.
<b>MATERIALS REQUIRED</b>	Pencil, eraser.
<b>LESSON OBJECTIVES</b>	After completing this lesson, you should be able to:  4-1. Identify common metric system terminology dealing with length, weight/mass, volume, area, and temperature, to include the meanings of metric prefixes.  4-2. Perform conversions within the metric system.  4-3. Convert from the U.S. Customary System of length, weight/mass, volume, area, and temperature to the metric system.  4-4. Convert from the metric system of length, weight/mass, volume, area, and temperature to the U.S. Customary System.
<b>SUGGESTION</b>	Work the following exercises (numbered frames) in numerical order. Write the answer in the space provided in the frame. After you have completed a frame, check your answer against solution given in the shaded area of the following frame. The final frame contains review exercises for Lesson 4. These exercises will help you to achieve the lesson objectives.

---

### FRAME 4-1.

**HISTORY.** Man has realized for thousands of years that he must have a system of weights and measures for trade and commerce. Ancient civilizations in Egypt, Mesopotamia, and the Indus Valley developed standard weights and measures. In the 13th century, England developed standards based on the older Roman standards. In 1789, the revolutionary government of France commissioned the French Academy of Science to establish a system of measurement and weights suitable for use throughout the world. The Academy created a system that was simple and scientific. The unit of length (meter or *metre*) was standardized by marking the distance on a platinum bar. Measures for capacity (volume) and mass (weight) were derived from the unit of length, thus relating the basic units of the system to each other. Furthermore, larger and smaller versions of each unit were created by multiplying or dividing the basic unit by 10 or multiples of 10, thus making this system (called the metric system) a "base 10" or "decimal" system. In 1799, these standards were legally adopted as the weights and measures in France.

What country is credited with developing a system of measurements founded upon the powers of 10?

---

The British Empire, however, did not adapt the metric system. Since the primary trading partners of the United States were Great Britain and Canada, the U.S. kept the "English" standards even though the U.S. had gone to a decimal system of coinage in 1786. In 1816, President Madison suggested going to the metric system, but the U.S. stayed with the English system.

Solution to  
Frame 4-1.

France

The metric system continued to gain in acceptance throughout the world. In 1866, the metric system was made legal in the United States. Eventually, the U.S. defined its "English" units in terms of the metric system. For example, one inch is defined as being equal to exactly 2.54 centimeters.

Many scientists believed that the metric system should be based upon natural standards of even greater permanence and greater precision. In 1960, the metric system underwent revision to become the International System of Units, usually called the SI (*Système International*). Among the changes made was that the meter was defined in wavelengths of a certain type of light. In 1983, the meter was again redefined to improve its accuracy. Now the meter is defined as the distance light in vacuum travels in  $1/299,792,458$  seconds. Although the technical definition has changed, the actual length of a meter remained unchanged.

How far would light in a vacuum travel in exactly one second?

\_\_\_\_\_ meters.

---

**FRAME 4-3.**

Currently, the United States is the only major country in the world to use the old "English" system (now usually called the United States Customary System) instead of the SI standard. The Metric Conversion Act of 1975 passed by the United States Congress states that "the policy of the United States shall be to coordinate and plan the increasing use of the metric system in the United States." The United States continues its conversion to the metric (SI) system (liter bottles of soft drinks replacing quart bottles, car engine displacement measured in liters instead of cubic inches, etc.).

Solution to  
Frame 4-2.

299,792,458 meters

The United States Customary System units are defined based on \_\_\_\_\_ units.

---

#### FRAME 4-4.

**BASIC METRIC MEASURES.** Under the United States Customary System of measurement, the inch, foot, or yard is used to measure length, the pound is used to measure weight, and the gallon is used to measure volume. In the SI or metric system, you would use meters for length, grams for mass, and liters for volume.

When using the metric system:

Length is expressed in \_\_\_\_\_.

Mass is expressed in \_\_\_\_\_.

Liquid capacity is expressed in \_\_\_\_\_.

Solution to  
Frame 4-3.

SI (or metric)

---

#### FRAME 4-5.

**NOTE:** In the remainder of this lesson, the term "metric" will be used to denote the SI system of measures.

Notice that when the U.S. system was discussed in Frame 4-4, the term "weight" was used; but when the metric system was discussed, the term "mass" was used. "Weight" measures gravity's attraction to a given object (its "heaviness"). Mass is a measure of an object's resistance to acceleration (its inertia). In other words, mass is a measure of how much matter is in the object while weight measures the force exerted by the object. For our purposes, we can say that weight and mass are the same. An object with a mass of 40 kilograms (40,000 grams), for example, will weight the same anywhere on the surface of the earth since the earth's gravity exerts the same pull. This works as long as you are dealing with the earth's gravity, but what happens if you are not? An object with a mass of 40 kilograms weighs about 88 pounds on earth. On the moon, the same object would weight about 15 pounds since the moon's gravitational pull is only one-sixth that of the earth's gravity. The object's mass, however, would remain unchanged (40 kilograms), but it would feel as heavy as a 6.7 kilogram weight on earth. In orbit around the earth, the object would be weightless (zero pounds), but still retain its mass (inertia) of 40 kilograms.

**NOTE:** In the U.S. system, the unit used to measure **mass** is the **slug** (about 14,594 grams).

In scientific matters, it is usually easier to speak of an object's \_\_\_\_\_ rather than its weight since its \_\_\_\_\_ does not change. (Einstein's theories of relativity are not considered in this subcourse.)

**NOTE:** For the remainder of this lesson, there will be no distinction between "weight" and "mass."

---

Solution to  
Frame 4-4.

meters  
grams  
liters

**FRAME 4-6.**

Meter. The term "metric" comes from *metre* (American spelling: meter), the unit of length. This term was derived from the Greek word *metron* (to measure). As previously stated, a meter is defined as the distance light travels in a vacuum in one 299,792,458th of a second (about 1.1 yard).

Solution to  
Frame 4-5.

mass  
mass

In the metric system, the basic unit of length is the \_\_\_\_\_ ,  
which is a little longer than the yard of the U.S. system.

---

**FRAME 4-7.**

Liter. The liter is equal to the volume of a cube measuring one decimeter (1/10 of a meter) on each side. A liter is equal to about 1.06 liquid quarts.

Solution to  
Frame 4-6.

meter

In the metric system, the basic unit of volume is the \_\_\_\_\_ ,  
which is a little more than the quart of the U.S. system.

---

**FRAME 4-8.**

Gram. The gram is the mass of one cubic centimeter (a cube measuring 1/100 meter on each side) of pure water at 4 degrees Celsius (about 39 degrees Fahrenheit) at sea level. This temperature is used because water has the highest concentration (density) at this temperature. Sea level ensures a stable gravitational pull and atmospheric pressure. A gram is equal to about 0.0022 pounds (about 454 grams to the pound)

Solution to  
Frame 4-7.

liter

In the metric system, the basic unit of mass (weight) is the \_\_\_\_\_ ,  
which is a little more than 1/30 of an ounce of the U.S. system.

---

**FRAME 4-9.**

**PREFIXES AND ROOT WORDS.** The great advantage of the metric system over the U.S. system is the metric use of root (basic) terms and standard prefixes. Meter, liter, and gram are examples of root words. They basically tell you what you are dealing with (length, volume, or weight). Prefixes (word parts that go in front) are added to the root word to denote how much.

Remember that the metric system is based on the decimal system (powers of ten). The prefix, then, denotes a power of ten. You have already come across some of these terms. In Frame 4-5, for example, the term "kilogram" was used.

In the word "kilogram," the root word (what) is \_\_\_\_\_  
and the prefix (how much) is \_\_\_\_\_ .

---

Solution to  
Frame 4-8.

gram

**FRAME 4-10.**

The prefix "kilo-" means "1000."

Therefore, something that weighs one kilogram weighs how many grams? \_\_\_\_\_

---

Solution to  
Frame 4-9.

root: gram  
prefix: kilo-

**FRAME 4-11.**

Some of the metric prefixes are given below. They denote larger and larger multiples of 10.

<u>Prefix</u>	<u>Meaning</u>	<u>Example</u>
deca-	10	1 decameter equals ten meters
hecto-	100	1 hectometer equals one hundred meters
kilo-	1,000	1 kilometer equals one thousand meters
mega-	1,000,000	1 megameter equals one million meters
giga-	1,000,000,000	1 gigameter equals one billion meters

How many meters are in 5.4 kilometers? \_\_\_\_\_

---

Solution to  
Frame 4-10.

1000 grams

**FRAME 4-12.**

All of the prefixes given in Frame 4-11 are multiples of 10. The metric system also used the negative powers of 10 (fractions whose **denominators** are multiples of 10) to denote smaller and smaller measurements. Some of these prefixes are given below.

Solution to  
Frame 4-11.

5,400 meters

<u>Prefix</u>	<u>Meaning</u>	<u>Example</u>
deci-	1/10	ten decimeters equal 1 meter
centi-	1/100	one hundred centimeters equal 1 meter
milli-	1/1000	one thousand millimeters equal 1 meter
micro-	1/1,000,000	one million micrometers equal 1 meter
nano-	1/1,000,000,000	one billion nanometers equal 1 meter

How many millimeters are in 3.34 meters? \_\_\_\_\_

---

**FRAME 4-13.**

As you can see, all that is needed to change kilometers to meters is to multiply by the number of meters in a kilometer (1000). This is done simply by moving the decimal point three places to the right. You can also convert meters to kilometers by dividing by 1000 (moving the decimal point three places to the left). Consider how simple this is compared to the U.S. Customary System. For example:

Solution to  
Frame 4-12.

3,340 millimeters

12 inches = 1 foot

3 feet = 1 yard

5.5 yards = 1 rod

40 rods = 1 furlong

8 furlongs = 1 mile

How many inches are there in one mile? \_\_\_\_\_

---

**FRAME 4-14.**

How many centimeters are in one kilometer? \_\_\_\_\_

Solution to  
Frame 4-13.

$12 \times 3 \times 5.5 \times 40 \times 8 = 63,360$   
inches

**FRAME 4-15.**

Originally, the meter was based on the polar circumference of the earth with the measurement going through Paris. The meter was to be one ten-millionth ( $1/10,000,000$ ) of the distance from the equator to the North Pole. If the Academy's measurements were correct, the distance from the equator to the North Pole would be ten \_\_\_\_\_ .  
(Hint: Refer to Frame 4-11.)

Solution to  
Frame 4-14.

$100 \times 1000 = 100,000$   
centimeters

(number of centimeters in a  
meter times number of  
meters in a kilometer)

NOTE: The polar circumference of the earth is estimated to be 40,000,008 meters. The distance from the equator to the North Pole is  $1/4$  the polar circumference, or about 10,000,002 meters.

NOTE: The earth is not a perfect sphere. The circumference of the earth measured around the equator is around 40,075,160 meters.

**FRAME 4-16.**

**UNITS OF DISTANCE.** In the metric system, the most commonly used measurements of distance are the meter, kilometer, centimeter, and millimeter.

Solution to  
Frame 4-15.

megameters

Since one millimeter equals 0.001 meter and one centimeter equals 0.01 meters, how many millimeters are in one centimeter? \_\_\_\_\_

**FRAME 4-17.**

As a quick review

10 millimeters = 1 centimeter.

100 centimeters = 1 meter

1000 meters = 1 kilometer

The abbreviation for meter is "m."

The abbreviation for kilometer is "km."

The abbreviation for centimeter is "cm."

The abbreviation for millimeter is "mm."

Fill in the blanks on the following chart

Millimeters	Centimeters	Meters	Kilometers
37 mm	= _____	= _____	= _____
_____	= 25 cm	= _____	= _____
_____	= _____	= 1.2 m	= _____
_____	= _____	= _____	= 3.3 km

---

Solution to  
Frame 4-16.

10

**FRAME 4-18.**

In the U.S. system, the normal units of length measurement are the inch, foot, yard, and mile. These units were defined in Frame 4-13. The approximate equivalents of these units in the metric system are given below. More exact conversion figures are given in the appendix.

$$1 \text{ inch} = 2.5 \text{ cm} = 25 \text{ mm}$$

$$1 \text{ foot} = 30 \text{ cm} = 0.3 \text{ m}$$

$$1 \text{ yard} = 91 \text{ cm} = 0.91 \text{ m}$$

$$1 \text{ mile} = 1.6 \text{ km}$$

A board is 2 1/2 feet long. How long is the board in metric units?

---

**FRAME 4-19.**

In the previous frame, you were given conversion information for changing a length measurement from the U.S. system to the metric system. The information below will help you to change lengths from the metric system to the U.S. system. More exact conversion figures are given in the appendix.

$$1 \text{ mm} = 0.04 \text{ inches}$$

$$1 \text{ cm} = 0.4 \text{ inches} = 0.03 \text{ feet}$$

$$1 \text{ m} = 1.1 \text{ yards} = 3.3 \text{ feet} = 39 \text{ inches}$$

$$1 \text{ km} = 0.62 \text{ (about } 5/8) \text{ miles} = 1094 \text{ yards} = 3281 \text{ feet}$$

A person runs a 1600-meter race. How far did he run in U.S. measure?

---

Solution to  
Frame 4-17.

$$37 \text{ mm} = 3.7 \text{ cm} = 0.037 \text{ m} = 0.000037 \text{ km}$$

$$250 \text{ mm} = 25 \text{ cm} = 0.25 \text{ m} = 0.00025 \text{ km}$$

$$1200 \text{ mm} = 120 \text{ cm} = 1.2 \text{ m} = 0.0012 \text{ km}$$

$$3,300,000 \text{ mm} = 330,000 \text{ cm} = 3,300 \text{ m} = 3.3 \text{ km}$$

Solution to  
Frame 4-18.

$$75 \text{ cm or } 3/4 \text{ (0.75) m}$$

(using the conversion formulas shown in Frame 4-18)

**FRAME 4-20.**

Jack ran a 100-yard race while Jill ran a 100-meter race.  
Who ran the longer race? \_\_\_\_\_

By how much? \_\_\_\_\_

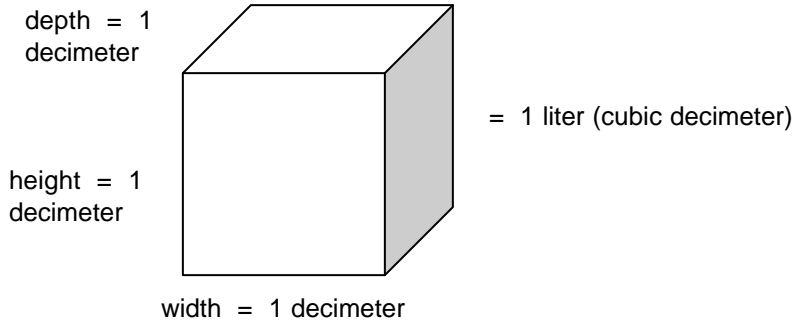
Solution to  
Frame 4-19.

1 mile (actually, a little less than a mile since a mile is about 1609 meters)

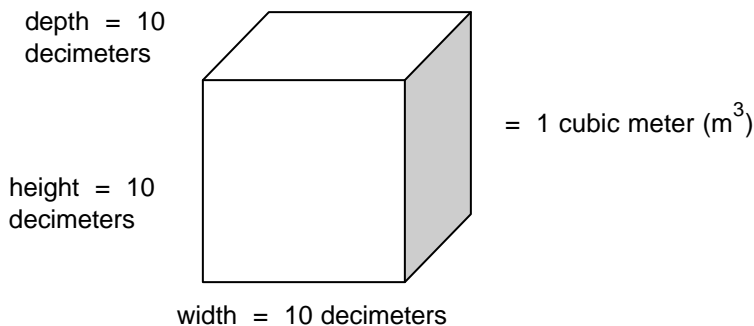
**FRAME 4-21.**

**UNITS OF VOLUME.** In the metric system, the most commonly used measurements of volume (capacity) are the liter, milliliter, cubic centimeter, decaliter, and cubic meter.

A liter is equal to the volume of a cube measuring one decimeter (1/10 of a meter) on each side. The volume of a cube is found by multiplying the length of one side by itself three times (width x height x depth). Since a liter is a volume equal to 1 decimeter times 1 decimeter times 1 decimeter, a liter can also be referred to as a cubic decimeter.



Since a decimeter is 1/10 of a meter, how much larger is a cubic meter than a cubic decimeter (1 liter)? \_\_\_\_\_  
(Hint: 1 meter = 10 decimeters)



Solution to  
Frame 4-20.

Jill

about 10 yards/9 m  
(100 m = 110 yards. Jill ran 110 yards  
OR  
100 yards = 91 m. Jack still needs to run 9 meters.)

**FRAME 4-22.**

Since there are 1000 cubic decimeters in a cubic meter and a liter is equal to one cubic decimeter, a cubic meter contains \_\_\_\_\_ liters.

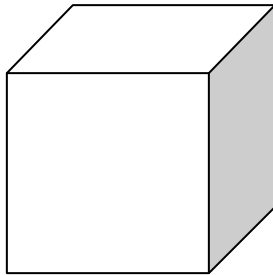
Solution to  
Frame 4-21.

1000 (10x10x10)  
(1000 cubic decimeters = 1  
 $m^3$ )

**FRAME 4-23.**

Ten centimeters are equal to one decimeter. How many cubic centimeters (cc) are in one cubic decimeter? \_\_\_\_\_

depth = 10  
centimeters



= 1 cubic decimeter

height = 10  
centimeters

width = 10 centimeters

Solution to  
Frame 4-22.

1000

**FRAME 4-24.**

There are 1000 cubic centimeters in one cubic decimeter.

A liter equals one cubic decimeter.

A milliliter is 1/1000 of a liter (see definition of "milli-" in Frame 4-12).

Therefore, a cubic centimeter equals \_\_\_\_\_ milliliter(s).

Solution to  
Frame 4-23.

1000  
(1000 cc = 1 cubic  
decimeter)

**FRAME 4-25.**

A cubic centimeter is abbreviated "cc."

A cubic meter is abbreviated "m<sup>3</sup>."

A liter is usually abbreviated "L."

A milliliter is usually abbreviated "mL."

NOTE: The term "liter" can also be abbreviated as a lower case letter. The upper case is used in this subcourse and elsewhere to help distinguish the letter "l" from the number "1."

A kiloliter (kL) is equal to \_\_\_\_\_.

---

Solution to  
Frame 4-24.

1 cc = 1 milliliter (mL)

**FRAME 4-26.**

In the U.S. system, a gallon equals four quarts. A quart is a little smaller than a liter. About how much is a gallon of gasoline when measured in the metric system?

\_\_\_\_\_

---

Solution to  
Frame 4-25.

1 cubic meter (m<sup>3</sup>)  
(1000 liters)

**FRAME 4-27.**

An inch equals 2.54 centimeters (cm).

There are \_\_\_\_\_ cubic centimeters in a cubic inch.

There are 1000 cubic centimeters in a liter.

How many cubic inches are in one liter? \_\_\_\_\_

---

Solution to  
Frame 4-26.

a little less than 4 liters

**FRAME 4-28.**

Sand, gravel, concrete, and similar commodities are often sold by the cubic yard. If you purchased a cubic yard of sand, you would be getting about \_\_\_\_\_ cubic meters of sand.

NOTE: A yard is about 0.91 meters.

Solution to  
Frame 4-27.

**16.4 cc (rounded)**  
(2.54 x 2.54 x 2.54 =  
16.387064)

about **61**  
(1000cc/16.4 cc/in<sup>3</sup>)

---

**FRAME 4-29.**

Fill in the blanks on the following chart.

Milliliters		Liters		Cubic meters
43 mL	=	_____	=	_____
_____	=	12 L	=	_____
_____	=	_____	=	3 m <sup>3</sup>

Solution to  
Frame 4-28.

**3/4 (0.75) m<sup>3</sup>** (rounded)  
(0.91 x 0.91 x 0.91)

**FRAME 4-30.**

The following shows a conversion chart for U.S. and metric volume measures.

Cubic inches	Cubic feet	Cubic yards	Metric measure
1	0.0006	0.00002	16.4 cc (or mL)
1,728	1	0.037	28.3 L
46,656	27	1	0.765 m <sup>3</sup>
0.061			1 cc
61	0.035		1 L
	35.3	1.308	1 m <sup>3</sup>

Solution to  
Frame 4-29.

43 mL = **0.043 L** =  
**0.000043 m<sup>3</sup>**

**12,000 mL** = 12 L =  
**0.012 m<sup>3</sup>**

**3,000,000 mL** =  
**3,000 L** = **.3 m<sup>3</sup>**

100 cubic inches equals about \_\_\_\_\_ (metric measure).

**FRAME 4-31.**

The following shows a conversion chart using U.S. liquid volume measures. Cubic inches is included to provide for conversion from U.S. customary liquid measures to U.S. customary measures.

<u>Fluid Ounce</u>	<u>Pint</u>	<u>Quart</u>	<u>Gallon</u>	<u>Cubic inch</u>	<u>Metric</u>
1	0.0625	0.03125	0.0078125	1.80	30 mL
16	1	0.5	0.125	28.88	0.47 L
32	2	1	0.25	57.75	0.95 L
128	8	4	1	231	3.79 L

Solution to  
Frame 4-30.

**1.64 L**  
16.4 mL x 100 =  
1640 mL = 1.64 L

A half-gallon of milk is equal to about \_\_\_\_\_ (metric measure).

---

**FRAME 4-32.**

**UNITS OF MASS (WEIGHT).** Four major metric units of mass are:

milligram (mg) -- 1/1,000 gram

gram (g)

kilogram (kg) -- 1,000 grams

metric ton -- 1,000 kg or 1 megagram

The milligram is so light (about the weight of a grain of sugar) that it is seldom used except in medicine and other scientific areas. The megagram or metric ton (about 2205 pounds) is used for heavy things. The metric ton is about 10 percent heavier than the U.S. short ton (2,000 pounds), but a little lighter than the U.S. long ton (2,240 pounds).

- a. An object weight 350 milligrams. It weighs \_\_\_\_\_ grams.
  - b. An object weight 1285 grams. It weighs \_\_\_\_\_ kilograms.
  - c. An object weighs a quarter of a kilogram. How many grams does it weigh? \_\_\_\_\_
  - d. How many grams are in a metric ton? \_\_\_\_\_
  - e. How many pounds (to the nearest 100 pounds) are in a metric ton? \_\_\_\_\_
  - f. How many U.S. short tons are in an U.S. long ton? \_\_\_\_\_
- 

Solution to  
Frame 4-31.

1.9 liters

**FRAME 4-33.**

The following chart shows conversions using U.S. weights. The weights are the avoirdupois system, which is the common weighing system for commerce. There are other systems of weights such as the apothecaries' (for pharmacy) and troy (used for precious metals such as gold). The avoirdupois, apothecaries' weight, and troy weight systems are based upon the grain, which is the same in all three systems. In the avoirdupois system, a pound equals 7,000 grains and is divided into 16 ounces. In the apothecaries' and troy systems, a pound equals 5,760 grains and is divided into 12 ounces.

***Avoirdupois***

<u>Grain</u>	<u>Ounce</u>	<u>Pound</u>	<u>Metric</u>
1	0.0023	0.00014	64.8 mg
437.5	1	0.0625	28.35 g
7,000	16	1	454 g (0.454 kg)

***Apothecaries' System of Weights***

1 grain = 64.8 milligrams

20 grains = 1 scruple (1.3 grams)

3 scruples = 1 dram (3.9 grams)

8 drams = 1 ounce (31.1 grams)

12 ounces = 1 pound (373 grams)

***Troy System of Weights***

1 grain = 64.8 mg

480 grains = 1 ounce (31.1 grams)

12 ounces = 1 pound (373 grams)

- Which is heavier, an ounce of iron (avoirdupois weight) or a ounce of gold (troy weight)? \_\_\_\_\_
- Which is heavier, a pound of iron (avoirdupois weight) or a pound of gold (troy weight)? \_\_\_\_\_

**NOTE:** The U.S. weights used in Frame 4-32 are the common avoirdupois weights.

**NOTE:** In this subcourse, the avoirdupois system of weights is used when referring to the U.S. Customary System of weights.

Solution to  
Frame 4-32.

- 0.35
- 1.285
- 250
- 1,000,000
- 2200
- 1.12

**FRAME 4-34.**

The following shows a conversion chart for changing from the U.S. Customary System of weights to metric units (values are approximate).

1 grain = 64.8 milligrams

1 ounce = 28.4 grams

1 pound = 454 grams or 0.454 kilograms

1 hundredweight = 100 pounds = 45.4 kilograms

1 ton = 20 hundredweight = 907 kilograms = 0.907 metric tons

Convert the following U.S. weights to metric units (round to nearest tenth).

a. 3 pounds = about \_\_\_\_\_ grams

b. 7 pounds = about \_\_\_\_\_ kilograms

c. 3 ounces = about \_\_\_\_\_ grams

d. 7.5 tons (short) = about \_\_\_\_\_ metric tons

---

Solution to  
Frame 4-33.

a. gold

b. iron

**FRAME 4-35.**

The following shows a conversion chart for changing from the metric system to U.S. Customary System (values are approximate).

1 milligram = 0.000035 ounces = 0.015 grains

1 gram = 0.035 ounces = 15 grains

1 hectogram = 100 grams = 3.5 ounces = 0.22 pounds

1 kilogram = 2.2 pounds

1 metric ton = 2205 pounds = 1.1 short tons

Convert the following metric measurements to U.S. Customary Standard units of weight.

- a. 2.5 metric tons = \_\_\_\_\_ tons (short)
- b. 650 grams = \_\_\_\_\_ ounces
- c. 4 kilograms = \_\_\_\_\_ pounds
- d. 1 kilogram = \_\_\_\_\_ ounces

Solution to  
Frame 4-34.

- a. 1362 g
- b. 3.2 kg (3.178)
- c. 85.2 g
- d. 6.8 t (6.8025)

---

**FRAME 4-36.**

**UNITS OF AREA.** Metric units for measuring area include the square centimeter (1 cm by 1 cm), the square meter (1 meter by 1 meter), the are (10 meters by 10 meters), the hectare (100 meters by 100 meters), and the square kilometer (1000 meters by 1000 meters).

1 square meter = 10,000 square centimeters = 1 centare (0.01 are)

1 are = 100 square meters

1 hectare = 100 are = 10,000 square meters

1 square kilometer = 100 hectare = 10,000 are = 1,000,000 sq. meters

An area of 12 square kilometers contains \_\_\_\_\_ hectares

Solution to  
Frame 4-35.

- a. 2.75 tons
- b. 22.75 ounces
- c. 8.8 pounds
- d. 35.2 ounces  
(2.2 x 16)

**FRAME 4-37.**

In the U.S. Customary System, area is measured using the square inch, square foot, square yard, acre, or square mile.

1 square foot = 144 square inches

1 square yard = 9 square feet = 1296 square inches

1 acre = 4,840 square yards = 43,560 square feet

1 square mile = 640 acres

An area of land measuring 70 yards by 70 yards is equal to a little more than one \_\_\_\_\_.

---

Solution to  
Frame 4-36

1,200

**FRAME 4-38.**

Information for converting from U.S. Customary System units to metric units is given below. Equivalents are approximate.

1 square inch = 6.5 square centimeters

1 square foot = 0.093 square meters

1 square yard = 0.836 square meters

1 acre = 4,047 square meters = 40.5 ares = 0.4 hectare

1 square mile = 2.59 square kilometers = 259 hectare

A piece of land one furlong (1/8 mile) square contains \_\_\_\_\_ acres or about \_\_\_\_\_ hectares.

---

Solution to  
Frame 4-37

acre  
(70 x 70 = 4900 square  
yards)

**FRAME 4-39.**

Information for converting from the metric system to the U.S. Customary System units of area is given below. Equivalents are approximate.

- 1 square centimeter = 0.155 square inches
- 1 square meter = 10.8 square feet = 1.2 square yards
- 1 are = 0.025 acres = 120 square yards
- 1 hectare = 2.48 acres
- 1 square kilometers = 0.386 square miles

A piece of land one kilometer by one kilometer contains about \_\_\_\_\_ acres.

---

Solution to  
Frame 4-38

10 acres  
( $1/8 \times 1/8 = 1/64$   
 $640 \times 1/64 = 10$ )  
4 hectare  
( $259/64 = 4.04688$   
or  
 $10/2.48 = 4.03226$ )

**FRAME 4-40.**

**TEMPERATURE.** Americans are familiar with the Fahrenheit scale developed by Gabriel Fahrenheit in the 18th century. In the Fahrenheit scale, water freezes at 32 degrees Fahrenheit ( $^{\circ}\text{F}$ ) and boils at  $212^{\circ}\text{F}$ . Shortly after Fahrenheit's scale was adopted, Anders Celsius, a Swede, developed a scale in which the freezing point of water is zero ( $0^{\circ}\text{C}$ ) and the boiling point of water is 100 ( $100^{\circ}\text{C}$ ). The Celsius scale was adopted by the metric system because of the convenience of the scale. Sometimes the Celsius scale is called the "centigrade" scale because one degree is one-hundredth (centi-) of the measurement between freezing and boiling.

Two primary points to remember when working with the Celsius scale are:

Water freezes at \_\_\_\_\_  $^{\circ}\text{C}$  and boils at \_\_\_\_\_  $^{\circ}\text{C}$ .

---

Solution to  
Frame 4-39

247 acres  
( $640 \times 0.386 = 247.04$ )

**FRAME 4-41.**

The number of degrees between freezing and boiling water on the Fahrenheit scale is 180 ( $212^{\circ} - 32^{\circ}$ ) and the number of degrees between freezing and boiling water on the Celsius scale is 100 ( $100^{\circ} - 0^{\circ}$ ). This means that one degree on the Celsius scale is equal to 1.8 degrees on the Fahrenheit scale ( $180/100 = 9/5 = 1.8$ ). Likewise, one degree on the Fahrenheit scale is equal to 0.5556 degrees (rounded to nearest ten-thousandth) on the Celsius scale ( $100/180 = 5/9 = 0.5555555...$ ).

Solution to  
Frame 4-40

$0^{\circ}\text{C}$   
 $100^{\circ}\text{C}$

Five degrees on the Celsius scale is equal to \_\_\_\_\_ degrees on the Fahrenheit scale.

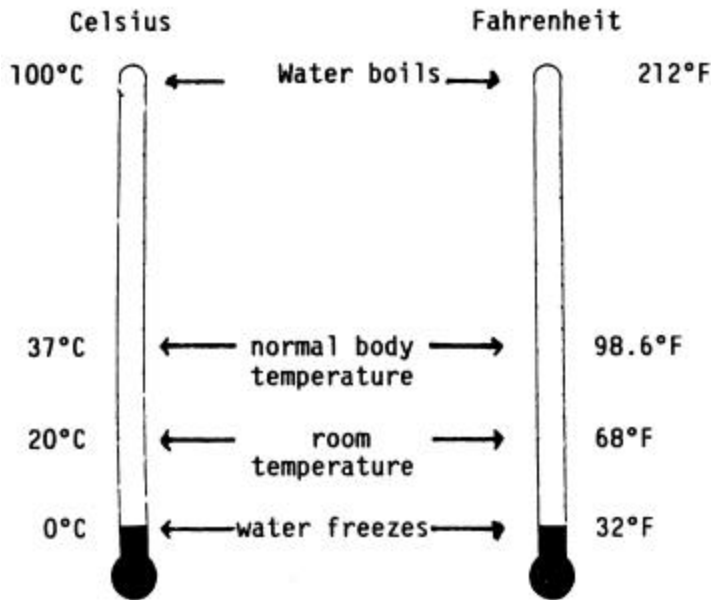
---

**FRAME 4-42.**

A quick comparison of the Celsius and Fahrenheit scales is shown below.

Solution to  
Frame 4-41

nine (9)



If you have average body temperature, your oral temperature is

\_\_\_\_\_  $^{\circ}\text{C}$ , which is the same as \_\_\_\_\_  $^{\circ}\text{F}$ .

---

**FRAME 4-43.**

Since  $1^{\circ}\text{C} = 1.8^{\circ}\text{F}$ , it appears that to change a Celsius temperature reading to a Fahrenheit temperature reading, you would simply multiply the Celsius temperature by 1.8. For example,  $20^{\circ}\text{C} \times 1.8 = 36^{\circ}\text{F}$ . If you look at the thermometers in Frame 4-42, however, you will see this is not so. What you have actually found is that  $20^{\circ}\text{C}$  above the freezing point of water is equal to  $36^{\circ}\text{F}$  above the freezing point of water. Since water freezes at  $32^{\circ}\text{F}$  on the Fahrenheit scale, 36 degrees above freezing would be  $36^{\circ}\text{F} + 32^{\circ}\text{F}$ , which is  $68^{\circ}\text{F}$ .

Remember: When converting from Celsius to Fahrenheit or from Fahrenheit to Celsius, you must adjust for the different freezing temperatures.

The formula for converting from Celsius to Fahrenheit is

$$^{\circ}\text{F} = (^{\circ}\text{C} \times 1.8) + 32^{\circ} \quad \text{OR} \quad ^{\circ}\text{F} = \frac{9}{5}^{\circ}\text{C} + 32^{\circ}$$

When converting from Celsius to Fahrenheit, you multiply by \_\_\_\_\_, then add \_\_\_\_\_ to the product.

---

Solution to  
Frame 4-42

$37^{\circ}\text{C}$  (or  $37.0^{\circ}\text{C}$ )

$98.6^{\circ}\text{F}$

**FRAME 4-44.**

Remember, when converting from Celsius to Fahrenheit, you **multiply first, then add**.

Convert the following Celsius temperatures to Fahrenheit using either of the formulas given in Frame 4-41. The formulas are the same except one uses a decimal form (1.8) and the other uses a fraction form (9/5).

a.  $0^{\circ}\text{C} = \underline{\hspace{2cm}}^{\circ}\text{F}$

b.  $100^{\circ}\text{C} = \underline{\hspace{2cm}}^{\circ}\text{F}$

c.  $38^{\circ}\text{C} = \underline{\hspace{2cm}}^{\circ}\text{F}$

d.  $212^{\circ}\text{C} = \underline{\hspace{2cm}}^{\circ}\text{F}$

---

Solution to  
Frame 4-43

$\frac{9}{5}$  (or 1.8)

$32^{\circ}$

**FRAME 4-45.**

If you wish to convert from Fahrenheit to Celsius, you must also consider the difference in freezing temperatures. For example, to convert from 77°F to Celsius, you must first determine how many degrees above freezing 77°F really is. This means you must subtract 32°F first.  $77^{\circ} - 32^{\circ} = 45^{\circ}$ . Since  $1^{\circ}\text{F} = 5/9^{\circ}\text{C}$ , you can multiply 45 by either 5/9 or by 0.5556 (conversion factor rounded to nearest ten-thousandth). The result is 25, that is, 25 degrees Celsius above freezing. Since freezing is 0°C, no further adjustment needs to be made.  $77^{\circ}\text{F} = 25^{\circ}\text{C}$ . The following formulas can be used to convert from Fahrenheit to Celsius.

$$^{\circ}\text{C} = \frac{5(^{\circ}\text{F} - 32^{\circ})}{9} \quad \text{OR} \quad ^{\circ}\text{C} = 0.5556 (^{\circ}\text{F} - 32^{\circ})$$

Remember, when converting from Fahrenheit to Celsius, you \_\_\_\_\_ first, then multiply (or multiple and divide).

Solution to  
Frame 4-44

- a. 32 [0 x 9/5 = 0;  
0+32 = 32]
- b. 212 [100 x 9/5 =  
180; 180 +  
32 = 212]
- c. 100.4 [38 x 1.8=  
68.4; 68.4+  
32 =100.4]
- d. 413.6 [212 x 1.8 =  
381.6;  
381.6 + 32=  
413.6]

**FRAME 4-46.**

Convert the following Fahrenheit temperatures to Celsius using either of the formulas given in Frame 4-43.

- a. 32°F = \_\_\_\_\_ °C
- b. 100°F = \_\_\_\_\_ °C
- c. 212°F = \_\_\_\_\_ °C
- d. 0°F = \_\_\_\_\_ °C

Solution to  
Frame 4-45

subtract

**FRAME 4-47.**

Notice that the last problem has a negative answer. In the Celsius system,  $0^{\circ}\text{C}$  is the freezing point of pure water; temperatures above freezing are positive values, and temperatures below freezing (below  $0^{\circ}\text{C}$ ) are denoted by negative numbers (numbers with a negative or minus sign in front). Negative values will be discussed in Lesson 5.

Is an object that has a temperature of  $100^{\circ}\text{C}$  twice as hot as an object that has a temperature of  $50^{\circ}\text{C}$ ?

Is an object that has a temperature of  $100^{\circ}\text{F}$  twice as hot as an object that has a temperature of  $50^{\circ}\text{F}$ ?

The answer to both of the above questions must be, "No," because we know of temperatures that go below 0 on each scale. But scientists desired a system of measurement in which the temperature measured the heat energy of an object, beginning with no heat energy. They gave the term "absolute zero" to this temperature. In 1848, William Thomson (later Baron Kelvin of Largs) introduced the absolute temperature scale based upon the Celsius scale. In this thermodynamic scale of temperature, an object with a temperature of zero has no heat energy. This temperature is referred to a zero kelvin (0 K). The freezing point of water is 273.15 K and the boiling point of water is 373.15 K. In 1954, the kelvin scale was adopted as the SI standard.

NOTE: Originally, temperature was denoted in degrees Kelvin ( $^{\circ}\text{K}$ ), but was later changed to kelvin (K) without the degree symbol. When spelled out, kelvin is spelled without the capital letter. The abbreviation for kelvin remains a capital letter (K).

The temperature at which an object contains no heat energy is \_\_\_\_\_.

---

**FRAME 4-48.**

A temperature of absolute zero (0 K) on the Celsius scale is  $-273.15^{\circ}\text{C}$ . On the Fahrenheit scale, absolute zero is  $-459.67^{\circ}\text{F}$ .

- The temperature at which pure water freezes is \_\_\_\_\_ K.
  - The temperature at which pure water boils is \_\_\_\_\_ K.
  - Of the Fahrenheit, Celsius, and kelvin scales, which has/have no negative temperatures?
- 

Solution to  
Frame 4-46

a.  $0$  [ $32 - 32 = 0$   
 $0 \times 5/9 = 0$ ;

b.  $37.8$ . [ $100 - 32 =$   
 $68$ ;  $68 \times$   
 $0.5556 =$   
 $37.7808$ ]

c.  $100$  [ $212 - 32 = 180$   
 $180 \times 5/9 =$   
 $100$ ]

d.  $-17.8$  [ $0 - 32 =$   
 $-32$ ;  
 $-32 \times 5/9 =$   
 $-17.7778$ ]

Solution to  
Frame 4-47

0 K (zero kelvin)

**Turn Page for Self-Test**

**FRAME 4-49.**

**SELF TEST.** You have completed the section on the metric (SI) system, the U.S. Customary System, and converting between the two systems.

If you feel that you need more review, look over the appropriate frames again. Then work the following self-test exercises shown below and on the following page. The solutions are found on the page following the self-test.

Solution to  
Frame 4-48

- a. 273.15
- b. 373.15
- c. kelvin

1. What metric unit would be most useful for measuring the following?
  - a. The volume of a single dose of medicine \_\_\_\_\_
  - b. The volume of milk in a plastic jug an the supermarket \_\_\_\_\_
  - c. The cargo space of an aircraft \_\_\_\_\_
  - d. The length of the side of a house \_\_\_\_\_
  - e. The amount of land that the house is on \_\_\_\_\_
  - f. The weight of a turkey in the supermarket \_\_\_\_\_
  - g. The weight of a railroad car \_\_\_\_\_
  - h. The distance between two towns \_\_\_\_\_
  - i. The temperature of a room \_\_\_\_\_
  - j. The temperature of an object in a laboratory that is near absolute zero \_\_\_\_\_

Instructions for exercises 2 through 7. Match the prefixes listed in Column A with their corresponding values in Column B by writing the letter of the response in the blank.

<u>Column A</u>	<u>Column B</u>
_____ 2. milli =	a. 1/100
_____ 3. centi =	b. 1/10
_____ 4. deci =	c. 10
_____ 5. deka =	d. 1,000
_____ 6. hecto =	e. 1/1000
_____ 7. kilo =	f. 100

8. Change the measurement in Column A to the desired unit given in Column B.

<u>Column A</u>	<u>Column B</u>
a. 74 meters	_____ kilometers
b. 125 milliliters	_____ liters
c. 34 square meters	_____ ares
d. 400 grams	_____ kilograms
e. 3 milliliters	_____ cubic centimeters
f. 3.2 hecares	_____ square meters
g. 2 liters	_____ cubic decimeters
h. 2.40 meters	_____ centimeters

9. Change the U.S. Customary System measurement in Column A to the metric unit given in Column B. Round to the nearest tenth.

<u>Column A</u>	<u>Column B</u>
a. 7 feet	_____ meters
b. 5 pounds	_____ kilograms
c. 1.5 ounces	_____ grams
d. 1.5 quarts	_____ liters
e. 47 °F	_____ °C
f. 170 square feet	_____ square meters
g. 5 cubic inches	_____ cubic centimeters

10. Change the metric measurement in Column A to the U.S. Customary System unit given in Column B. Round to the nearest tenth.

<u>Column A</u>	<u>Column B</u>
a. 35 °C	_____ °F
b. 3.2 meters	_____ feet
c. 25 liters	_____ gallons
d. 12 kilograms	_____ pounds
e. 15 centimeters	_____ inches

***Check Your Answers on Next Page***

## SOLUTIONS TO FRAME 4-49 (SELF-TEST)

1.
  - a. milliliter (or cubic centimeter)
  - b. liter
  - c. cubic meter
  - d. meter
  - e. are (or hectare)
  - f. kilogram
  - g. metric ton
  - h. kilometer
  - i. degrees Celsius
  - j. kelvin
  
2. e
3. a
4. b
5. c
6. f
7. d
  
8.
  - a. 0.074 kilometers ( $74 \times 1/1000$ )
  - b. 0.125 liters (or  $1/8$  liter) ( $125 \times 1/1000$ )
  - c. 0.34 ares ( $34 \times 1/100$ )
  - d. 0.4 kilograms ( $400 \times 1/1000$ )
  - e. 3 cubic centimeters ( $1 \text{ mL} = 1 \text{ cc}$ )
  - f. 32,000 square meters ( $3.2 \text{ hectare} = 320 \text{ are} = 320 \times 100 \text{ square meters}$ )
  - g. 2 decimeters ( $1 \text{ liter} = 1 \text{ cubic decimeter}$ )
  - h. 240 centimeters ( $2.4 \times 100$ )
  
9.
  - a. 2.1 meters ( $7 \times 0.3$ )
  - b. 2.3 kilograms ( $5 \times 0.454 = 2.270$ )
  - c. 42.5 grams ( $1.5 \times 28.35$ )
  - d. 1.4 liters ( $1.5 \times 0.95 = 1.425$ )
  - e.  $8.3^\circ\text{C}$  ( $47 - 32 = 15$ ;  $15 \times 5/9 = 75/9 = 8.33333\dots$ )
  - f. 15.8 square meters ( $170 \times 0.093 = 15.81$ )
  - g. 82 cubic centimeters ( $5 \times 16.4$ )
  
10.
  - a.  $95^\circ\text{F}$  ( $35 \times 9/5 = 63$ ;  $63 + 32 = 95$ )
  - b. 10.6 feet ( $3.2 \times 3.3 = 10.56$ )
  - c. 6.6 gallons ( $25 \div 3.79 = 6.5963$ )
  - d. 26.4 pounds ( $12 \times 2.2$ )
  - e. 6 inches ( $15 \times 0.4$ ) Using appendix:  $15 \times 0.394 = 5.9$  inches

*End of Lesson 4*

## LESSON ASSIGNMENT

**LESSON 5** Negative Numbers, Scientific Notation, and Square Roots.

**LESSON ASSIGNMENT** Frames 5-1 through 5-33.

**MATERIALS REQUIRED** Pencil, eraser.

**LESSON OBJECTIVES** After completing this lesson, you should be able to:

- 5-1. Add, subtract, multiply, and divide using negative numbers.
- 5-2. Change numbers to and from scientific notation.
- 5-3. Multiply and divide using scientific notation.
- 5-4. Estimate square roots.
- 5-5. Identify the terms "absolute value," "square," and "square root."

**SUGGESTION** Work the following exercises (numbered frames) in numerical order. Write the answer in the space provided in the frame. After you have completed a frame, check your answer against solution given in the shaded area of the following frame. The final frame contains review exercises for Lesson 5. These exercises will help you to achieve the lesson objectives.

---

### FRAME 5-1.

**NEGATIVE NUMBERS.** In the previous lesson, one of your answers involved a temperature below zero. The answer was given as a negative number. Sometimes it is easier to understand negative values if you use a device such as the number line represented below. Positive numbers are to the right of the zero; negative numbers are to the left of the zero. (Zero is usually classified as a "non-negative" number.) Both ends of the number line extends to infinity (without end). All whole numbers, fractions, decimals, and mixed numbers are represented on the number line. Negative numbers are denoted by a negative or minus sign ( $-$ ) before the number. Positive numbers are denoted by a positive or plus sign ( $+$ ) before the number or by having no sign before the number.



On a number line, \_\_\_\_\_ numbers are usually to the left of zero while positive numbers are to the \_\_\_\_\_ of zero.

---

**FRAME 5-2.**

You have reviewed how to add, subtract, multiply, and divide positive numbers in the preceding lessons. This lesson will give you rules for adding, subtracting, multiplying, and dividing when negative numbers are involved.

One concept that is helpful when working with negative numbers is "absolute value." Absolute value pertains to the numerical value of a figure without regard to whether it is a positive or negative number. Using the number line, it is the number's distance from zero without regard whether it is to the right or left of the zero. Another way of think of absolute value is that positive numbers stay positive and negative numbers become positive. The symbol for absolute value is two parallel lines with the number between the lines.

The absolute value of negative eight can be written as  $|-8|$ .

If you add the absolute values of two numbers, you add the values of the numbers without regard to whether the numbers are positive or negative. For example

$$|-2| + |-3| = |+2| + |+3| = |+2| + |-3| = |-2| + |+3| = 5$$

The absolute value of negative five ( $|-5|$ ) is the same as the absolute value of \_\_\_\_\_.

---

**FRAME 5-3.**Rule for Addition of Two Positive Numbers

- (1) Change both numbers to absolute values.
- (2) Add their absolute values
- (3) Place a positive symbol (or no symbol) in front of the sum.

Add:  $+5 + +3$     Answer \_\_\_\_\_

---

Solution to  
Frame 5-1.

negative

right

Solution to  
Frame 5-2.

positive five ( $|+5|$ )

**FRAME 5-4.**

Rule for Addition of Two Negative Numbers

- (1) Change both numbers to absolute values.
- (2) Add their absolute values.
- (3) Place a negative symbol in front of the sum.

Add:  $-5 + -3$     Answer \_\_\_\_\_

---

Solution to  
Frame 5-3.

$$\begin{aligned} &+8 \\ +5 + +3 &= |+5| + |+3| = \\ +|5 + 3| &= +8 \end{aligned}$$

**FRAME 5-5.**

Rule for Addition of a Positive Number and a Negative Number

- (1) Change both numbers to absolute values.
- (2) Subtract the smaller absolute value from the larger absolute value.
- (3) Place the original sign of the larger absolute value in front of the difference (remainder).

a. Add:  $+5 + -3$     Answer \_\_\_\_\_

b. Add:  $-5 + +3$     Answer \_\_\_\_\_

---

Solution to  
Frame 5-4.

$$\begin{aligned} &-8 \\ -5 + -3 &= |-5| + |-3| = \\ -|5 + 3| &= -8 \end{aligned}$$

**FRAME 5-6.**

In exercise "a" above, did you notice that adding a "smaller" negative number to a positive number is much like subtracting a smaller positive number from a larger positive number?

Also, the answers to "a" and "b" had the same absolute value. The difference is that the first problem resulted in a positive answer and the second resulted in a negative answer.

Remember: **Add** the absolute values if the signs of the two numbers are the same (both positive, both negative) and \_\_\_\_\_ the absolute values if the signs of the two numbers are not the same (one positive, one negative).

Solution to  
Frame 5-5.

$$\begin{aligned} \text{a. } &+2 \\ +5 + -3 &= \\ +|+5| - |-3| &= \\ +|5 - 3| &= +2 \\ \text{b. } &-2 \\ -5 + +3 &= \\ -|-5| + |+3| &= \\ -|5 - 3| &= -2 \end{aligned}$$

---

**FRAME 5-7.**Rule for Subtraction

Change the sign of the subtrahend (second or bottom number), then add the two numbers according to the rules for addition given in Frames 5-3, 5-4, and 5-5.

Solution to  
Frame 5-6.

subtract

Subtract these numbers (both numbers are positive).

a. Subtract:  $+5 - +3$     Answer \_\_\_\_\_

b. Subtract:  $+3 - +5$     Answer \_\_\_\_\_

**FRAME 5-8.**

Subtract these numbers (both numbers are negative).

a. Subtract:  $-5 - -3$     Answer \_\_\_\_\_

b. Subtract:  $-3 - -5$     Answer \_\_\_\_\_

Solution to  
Frame 5-7.

a.  $+2$

$$+5 - +3 =$$

$$+5 + -3.$$

(Add pos and neg)

$$|+5| - |-3|$$

$$+|5 - 3| = +2$$

b.  $-2$

$$+3 - +5 =$$

$$+3 + -5.$$

(Add pos and neg)

$$|-5| - |+3|$$

$$-|5 - 3| = -2$$

**FRAME 5-9.**

Subtract these numbers (positive minus negative).

a. Subtract:  $+5 - ^{-}3$     Answer \_\_\_\_\_

b. Subtract:  $+3 - ^{-}5$     Answer \_\_\_\_\_

Solution to  
Frame 5-8.

a.  $^{-}2$

$$^{-}5 - ^{-}3 =$$

$$^{-}5 + +3.$$

(Add pos and neg)

$$|^{-}5| - |+3| =$$

$$^{-}|5 - 3| = ^{-}2$$

b.  $+2$

$$^{-}3 - ^{-}5 =$$

$$^{-}3 + +5.$$

(Add pos and neg)

$$|+5| - |^{-}3| =$$

$$+|5 - 3| = +2$$

**FRAME 5-10.**

Subtract these numbers (negative minus positive).

a. Subtract:  $^{-}5 - +3$     Answer \_\_\_\_\_

b. Subtract:  $^{-}3 - +5$     Answer \_\_\_\_\_

Solution to  
Frame 5-9.

a.  $+8$

$$+5 - ^{-}3 =$$

$$+5 + +3.$$

(Add pos and pos)

$$|+5| + |+3| =$$

$$+|5 + 3| = +8$$

b.  $+8$

$$+3 - ^{-}5 =$$

$$+3 + +5.$$

(Add pos and pos)

$$|+3| + |+5| =$$

$$+|3 + 5| = +8$$

**FRAME 5-11.**

Rule for Multiplication

- (1) Change both numbers to absolute values.
- (2) Multiply the absolute values.
- (3) Place the appropriate sign in front of the product.
  - (a) If the original numbers have the same sign (both positive or both negative), the sign of the product is positive.
  - (b) If the original numbers have different signs (one positive and one negative), the sign of the product is negative.

Solve these problems

- a. Multiply:  $+5 \times +3$     Answer \_\_\_\_\_
- b. Multiply:  $-5 \times +3$     Answer \_\_\_\_\_
- c. Multiply:  $+5 \times -3$     Answer \_\_\_\_\_
- d. Multiply:  $-5 \times -3$     Answer \_\_\_\_\_

Solution to  
Frame 5-10.

a.  $-8$

$$-5 - +3 =$$

$$-5 + -3.$$

(Add neg and neg)

$$|-5| + |-3|$$

$$-|5 + 3| = -8$$

b.  $-8$

$$-3 - +5 =$$

$$-3 + -5.$$

(Add neg and neg)

$$|-3| + |-5|$$

$$-|3 + 5| = -8$$

**FRAME 5-12.**

Rule for Division

- (1) Change both numbers to absolute values.
- (2) Divide the divisor into the dividend (just like both numbers were positive).
- (3) Place the appropriate sign in front of the quotient.
  - (a) If the original divisor and dividend have the same sign (both positive or both negative), the sign of the quotient is positive.
  - (b) If the original divisor and dividend have different signs (one positive and one negative), the sign of the quotient is negative.

Solution to  
Frame 5-11.

- a.  $+15$   
(pos x pos = pos)
- b.  $-15$   
(neg x pos = neg)
- c.  $-15$   
(pos x neg = neg)
- d.  $+15$   
(neg x neg = pos)

Solve the following problems.

- a.  $+8 \div +4$  Answer \_\_\_\_\_
  - b.  $-8 \div +4$  Answer \_\_\_\_\_
  - c.  $+8 \div -4$  Answer \_\_\_\_\_
  - d.  $-8 \div -4$  Answer \_\_\_\_\_
-

**FRAME 5-13.**

**SCIENTIFIC NOTATION.** When you are looking at a very large or very small number, one with a lot of zeroes, do you ever wish that someone would count the zeroes for you so you would know what the number is?

Well, there is a system that does that for you. It is called "scientific notation." Scientific notation is a method of writing numbers in terms of the powers of 10. You have already studied the powers of 10 in Lessons 1 and 4 of this subcourse.

Basically  $10^P$  (ten to the p-th power) is a "1" followed by "P" zeroes if the "P" is positive. If the "P" is negative, then "P" is the number of decimal places to the right of the decimal point [decimal point followed by (P-1) zeroes followed by the numeral one (1).]

NOTE: The superscripted (raised) P (" $P$ ") is called the "exponent."

a.  $10^7$  is written as \_\_\_\_\_

b.  $10^{-7}$  is written as \_\_\_\_\_

---

Solution to  
Frame 5-12.

a.  $+2$   
(pos + pos = pos)

b.  $-2$   
(neg + pos = neg)

c.  $-2$   
(pos + neg = neg)

d.  $+2$   
(neg + neg = pos)

**FRAME 5-14.**

Convert the following numbers to  $10^P$  format.

a. 10,000 \_\_\_\_\_

b. 0.00001 \_\_\_\_\_

---

Solution to  
Frame 5-13.

a. 10,000,000

b. 0.0000001

**FRAME 5-15.**

The above works for numbers that consist of one "1" and any number of zeroes, but how about other numbers like 32,700,000,000,000 or 0.0000000000065?

In scientific notation, a number is reduced to a standard form. That form is a number between 1 and 10 (the number can be a decimal such as 3.475) followed by "x 10<sup>P</sup>" with "P" being the power needed to restore the new number to the original number. The number before the "times" symbol (x) is sometimes called the "coefficient."

Remember: scientific notation begins with a positive single digit (1, 2, 3, 4, 5, 6, 7, 8, or 9) which may or may not be followed by a decimal point and additional digits.

For example: 32,700,000,000,000 can be converted into scientific notation.

Place a decimal point at the end of the number (following the zero in the unit's [one's] position).

Move the decimal point so that it falls after the "3" (the first non-zero digit of the original number starting from the left). Count the number of places you moved the decimal.

You moved the decimal point 13 places to the left to obtain 3.2700000000000. Therefore, the exponent "P" equals 13.

$$3.2700000000000 \times 10^{13} = 32,700,000,000,000 .$$

The zeros are usually dropped as long as no non-zero digit follows.  $32,700,000,000,000 = 3.27 \times 10^{13}$ . The scientific notation stands for the product of the following multiplication problem:  
 $3.27 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ .

Convert the following numbers to scientific notation.

a. 38,000,000 \_\_\_\_\_

b. 40,100 \_\_\_\_\_

Solution to  
Frame 5-14.

a.  $10^4$

b.  $10^{-5}$

In "b," did you remember to count the "1"? Did you remember to make the exponent negative?

**FRAME 5-16.**

In problem "b" of the previous frame, the zero between the four and the one is not dropped. Only zeroes following the last non-zero digit can be dropped without changing the value of the number.

However, you may want to round the number to make it easier to use; that is, you do not need a high degree of accuracy. For example,  $4.01 \times 10^4$  rounded to the nearest ten-thousand can be expressed as  $4 \times 10^4$ .

Note: If you rounded  $4.01 \times 10^4$  to the nearest thousand, you would also get  $4 \times 10^4$ .

- a. Change 3,756 to scientific notation. \_\_\_\_\_
- b. Change 3,756 to scientific notation rounded to the nearest thousand. \_\_\_\_\_  
(Hint: Round first, then change to scientific notation.)
- 

Solution to  
Frame 5-15.

- a.  $3.8 \times 10^7$
- b.  $4.01 \times 10^4$

**FRAME 5-17.**

The preceding frames work fine for large numbers, but how about small numbers, like. 0.000000000065?

Begin at the decimal point.

Move the decimal point to the right until it passes the first non-zero number (the "6"). Count the number of places you are moving the decimal.

You moved the decimal point 11 places to the right to obtain 6.5. Therefore "P" equals "-11." Remember, if you move the decimal point to the right, the exponent will be negative.

$$0.000000000065 = 6.5 \times 10^{-11} .$$

Convert the following numbers to scientific notation.

- a. 0.0072 \_\_\_\_\_
- b. 0.101 \_\_\_\_\_
- c. 3.9 \_\_\_\_\_

[Note: Remember that 10 (or any number) raised to the zero power is "1" and that any number times "1" remains the original number.]

---

Solution to  
Frame 5-16.

- a.  $3.756 \times 10^3$
- b.  $4 \times 10^3$

**FRAME 5-18.**

To convert a number in scientific notation back to normal, you move the decimal point based upon the power of 10 (the "P"). If the "P" is positive, move the decimal point "P" places to the right.

If the number in scientific notation form does not have a decimal point (that is, the coefficient is a whole single digit number), place the decimal point following the coefficient.

- a. Convert  $4.5 \times 10^4$  to normal format. \_\_\_\_\_
- b. Convert  $7 \times 10^6$  to normal format. \_\_\_\_\_
- 

Solution to  
Frame 5-17.

- a.  $7.2 \times 10^{-3}$
- b.  $1.01 \times 10^{-1}$
- c.  $3.9 \times 10^0$   
[ $3.9 \times 10^0 =$   
 $3.9 \times 1 = 3.9$ ]

**FRAME 5-19.**

If the "P" is negative, convert a number in scientific notation to normal format by moving the decimal point "P" places to the left.

- a. Convert  $4.5 \times 10^{-4}$  to normal format. \_\_\_\_\_
- b. Convert  $7 \times 10^{-6}$  to normal format. \_\_\_\_\_
- 

Solution to  
Frame 5-18.

- a. 45,000
- b. 7,000,000

**FRAME 5-20.**

Multiplying Using Scientific Notation

Scientific notation can make multiplication a little easier (or at least look neater).

Consider the problem:  $32,000 \times 10,200,000,000$ .

By using scientific notation, you can change the appearance of the problem to  $3.2 \times 10^4$  times  $1.02 \times 10^{10}$ .

To solve the problem, you

- (1) Multiply the coefficients (numbers in front).
- (2) **Add** the exponents (the powers of 10). The sum will be your new power of 10.
- (3) Rewrite the answer so it is in scientific notation format, if needed.

$$3.2 \times 10^4 \times 1.02 \times 10^{10} = 3.2 \times 1.02 \times 10^{4+10} = 3.264 \times 10^{14}$$

Solve this problem:

$$3 \times 10^5 \times 7 \times 10^6 \quad \underline{\hspace{2cm}}$$

Solution to  
Frame 5-19.

- a. 0.00045
- b. 0.000007

**FRAME 5-21.**

**NOTE:** When you multiplied the coefficients to the problem in Frame 5-20, you got a new coefficient that was above 10 ( $21 \times 10^{11}$ ). However, Frame 5-15 states that the coefficient should be between 1 and 10. That is, the number to the left of the decimal point is to be a single digit.

Since  $21 = 2.1 \times 10^1$   
Then  $21 \times 10^{11} = 2.1 \times 10^1 \times 10^{11} = 2.1 \times 10^{1+11} = 2.1 \times 10^{12}$ .

Solve these two problems:

a.  $3 \times 10^{-4}$  times  $2 \times 10^{-3} = \underline{\hspace{2cm}}$

b.  $5 \times 10^{-4}$  times  $2 \times 10^{-6} = \underline{\hspace{2cm}}$

**NOTE:** Positive and negative exponents are added using the same rules for adding positive and negative numbers.

Solution to  
Frame 5-20.

$21 \times 10^{11}$   
(or  $2.1 \times 10^{12}$ )

**FRAME 5-22.**

Dividing Using Scientific Notation

Scientific notation can also be used in division.

Consider the problem:  $800,000 \div 200,000,000$ .

By using scientific notation, you can change the appearance of the problem to  $8 \times 10^5$  divided by  $2 \times 10^8$ .

To solve the problem, you

- (1) Divide the coefficients (numbers in front).
- (2) **Subtract** the exponents. The difference will be your new power of 10.
- (3) Rewrite the answer so it is in scientific notation format, if needed.

$$8 \times 10^5 \div 2 \times 10^8 = 8 \div 2 \times 10^{5-8} = 4 \times 10^{-3} \text{ (or 0.004)}$$

Solve this problem:

$$5 \times 10^7 \div 4 \times 10^5 \text{ _____}$$

---

**FRAME 5-23.**

Solve these division problems. Remember to use the rules for subtracting negative numbers.

a.  $8 \times 10^7 \div 4 \times 10^{-5}$  \_\_\_\_\_

b.  $4 \times 10^{-7} \div 8 \times 10^{-5}$  \_\_\_\_\_

c.  $4 \times 10^{-7} \div 3 \times 10^3$  \_\_\_\_\_

(Hint: Round to the second place following the decimal.)

---

Solution to  
Frame 5-21.

a.  $6 \times 10^{-7}$

$(3 \times 2 \times 10^{-(4+3)})$

b.  $10^{-1}$  or 0.1

$5 \times 2 \times 10^{4-6}$

$10 \times 10^{-2}$

$1 \times 10^1 \times 10^{-2}$

$1 \times 10^{1-2}$

$10^{-1}$

Solution to  
Frame 5-22.

a.  $1.25 \times 10^2$  or 125

$5/4 \times 10^{7-5} =$

$1.25 \times 10^2$

**FRAME 5-24.**

**SQUARES AND SQUARE ROOTS**

Square

When a number is multiplied by itself, it is said to be "squared." For example, five times five equals twenty-five. This can be written as  $5 \times 5 = 25$  or as  $5^2 = 25$ . In the second method, the statement can be read as "five to the second power equals twenty-five" or as "five squared equals twenty-five."

The term "square" comes from the formula for determining the area of a square, which is  $s^2$  (the length of one side of the square multiplied by itself). To "square" a number, multiply the number by itself.

Square the following numbers:

- a. 15 \_\_\_\_\_
- b. 0.03 \_\_\_\_\_
- c.  $1/2$  \_\_\_\_\_

Solution to  
Frame 5-23.

a.  $2 \times 10^{12}$   
 $8/4 \times 10^{7-(-5)} =$   
 $2 \times 10^{7+5}$

b.  $5 \times 10^{-3}$   
 $4/8 \times 10^{-7-(-5)} =$   
 $0.5 \times 10^{-7+5} =$   
 $5 \times 10^{-1} \times 10^{-2}$   
 $= 5 \times 10^{-(1+2)}$

c.  $1.33 \times 10^{-10}$   
 $4/3 \times 10^{-7-(-3)} =$

$1.3333 \times 10^{-7+(-3)} =$

$1.3333 \times 10^{-(7+3)}$

**FRAME 5-25.**

Did you notice that the squares for "b" and "c" above are smaller than the original number?

For positive numbers, the "square" is larger than the original number if the original number is greater than 1 and is smaller than the original number if the original number is less than 1.

The square of a negative number will be a \_\_\_\_\_ number.

Solution to  
Frame 5-24.

a. 225

b. 0.0009

c.  $1/4$

**FRAME 5-26.**

Square Root

In some problems, you have a number and need to know what number squared equals that number. This is called "square root." For example, five squared is twenty-five; therefore, the square root of twenty-five is five.

This is about the same as saying that the area of a square is 25 square meters. What is the length of one side of the square?

The "square root" is the reversal of the square function.

The symbol for square root is  $\sqrt{\quad}$ .

"The square root of 25" is written as  $\sqrt{25}$ .

Another way of indicating square root is using "1/2" as the exponent (power of 10). For example:  $25^{1/2} = 5$ .

See if you know the square roots of the following numbers.

a. 625 \_\_\_\_\_

b. 0004 \_\_\_\_\_

c. 1/81 \_\_\_\_\_

[Hint: Take a guess, then square your guess. Adjust your guess higher or lower until you hit upon the answer.]

Solution to  
Frame 5-25.

positive

**FRAME 5-27.**

You may have been able to guess the square roots of the preceding problems, but you can see that this would not be a good way to determine the square root of numbers like 37,594,227.0374.

The easiest way of finding the square root of a number is to use a hand-held calculator or a computer. There is a method for calculating the square root of a number using paper and pencil, but this method takes time and will not be presented here.

Sometimes you may just want a good guess as to the square root or you may just want to know where the decimal point goes.

In such cases, it is useful to have a method of determining the approximate square root of a number. First, review the squares of the integers from 1 to 9. Remember, the square root of the square is the original number.

$1^2 = 1$	therefore	$1^{1/2} = 1$
$2^2 = 4$	therefore	$4^{1/2} = 2$
$3^2 = 9$	therefore	$9^{1/2} = 3$
$4^2 = 16$	therefore	$16^{1/2} = 4$
$5^2 = 25$	therefore	$25^{1/2} = 5$
$6^2 = 36$	therefore	$36^{1/2} = 6$
$7^2 = 49$	therefore	$49^{1/2} = 7$
$8^2 = 64$	therefore	$64^{1/2} = 8$
$9^2 = 81$	therefore	$81^{1/2} = 9$

**NOTE:** A number that is an exact square of an integer (whole number) is sometimes called a "perfect square." The square root of a perfect square is an integer.

If the square of 387 is 149,769, then the square root of 149,769

is \_\_\_\_\_ .

---

Solution to  
Frame 5-26

a. 25

b. 0.02

c. 1/9

## FRAME 5-28

### Estimating the Square Root of a Number Greater Than 1

The methods for estimating square root differs slightly depending upon whether the number is greater than 1 or less than 1. Let's begin with numbers that are greater than 1.

- (1) Pair off the digits of the number beginning at the decimal point (or where the decimal point would be if the number had one) and going to the left. Drop any digits to the right of the decimal point.
- (2) Identify the last digit or pair of digits (the digit or pair of digits at the beginning of the number). If the number had an even number of digits to the left of the decimal, you will have a pair of digits. If the number had an odd number of digits to the left of the decimal, you will have a single digit.
  - (a) If the digit/pair identified in step 2 is a perfect square (1, 4, 9, 16, 25, 36, 49, 64, or 81), replace the digit/pair with the square root of that digit/pair.
  - (b) If the digit/pair identified in step 2 is not a perfect square, identify the largest perfect square that is less than the digit/pair and replace the digit/pair with the square root of that perfect square.
- (3) For each **pair** of digits following the digit or digits identified in step 2, substitute a zero.
- (4) The resulting number is the estimated square root (low).
- (5) Increase the left (first) digit of the estimated square root (low) by 1 to arrive at the estimated square root (high).
- (6) The actual square root will be less than the estimated square root (high) and equal to or greater than the estimated square root (low).

**NOTE:** If the digit/pair identified in step 2 is a perfect square and the following pairs were all zeros originally and no non-zero digits followed the decimal point of the original number, then the estimated square root (low) is the actual square root.

Estimate the square root of the following numbers using the above rules.

- a. 149,769
- b. 640,000
- c. 36,000

Solution to  
Frame 5-27

387

### FRAME 5-29

In case you had difficulty with the square roots given in Frame 5-28, the problems are worked in greater detail below.

a. 149,769

(1) Pair off beginning at the decimal point  
 $149,769. = (14)(97)(69)$

(2) Largest perfect square not over 14 is 9. Square root of 9 is 3.

(3) Replace

(4) low  $(14)(97)(69) \rightarrow (3)(0)(0) \rightarrow 300$

(5) high  $(3+1)(0)(0) \rightarrow 400$

(6) The actual square root of 149,769 is more than 300 and less than 400. (See Frame 5-27)

b. 640,000

(1) Pair off beginning at the decimal point  
 $640,000. = (64)(00)(00)$

(2) Largest perfect square not over 64 is 64. Square root of 64 is 8.

(3) Replace

(4) low  $(64)(00)(00) \rightarrow (8)(0)(0) \rightarrow 800$

Stop calculations. Based upon the NOTE, 800 is the exact square root of 640,000.

c. 36,000

(1) Pair off beginning at the decimal point  
 $36,000. = (3)(60)(00)$

(2) Largest perfect square not over 3 is 1. Square root of 1 is 1.

(3) Replace

(4) low  $(3)(60)(00) \rightarrow (1)(0)(0) \rightarrow 100$

(5) high  $(1+1)(0)(0) \rightarrow 200$

(6) The actual square root of 36,000 is more than 100 and less than 200. (Were you tricked because the square root of 36 is 6? Remember, you must begin paring off starting at the decimal point.)

### Solution to Frame 5-28

a. between 300 and  
400  
(actual square root  
is 387)

b. 800 (exact)

c. between 100 and  
200  
(actual approximate  
square root is  
189.7366596...)

## FRAME 5-30

### Estimating the Square Root of a Number Less Than 1

The method below is for decimal numbers less than one. If you have a fraction, change the fraction to a decimal and use the procedure given below. There are methods for calculating the square root of a fraction, but they are not covered in this subcourse.

- (1) Pair off the digits of the number beginning at the decimal point and going to the **right**.
- (2) Identify the first digit **pair** with a non-zero digit.
  - (a) The identified pair must contain two digits, not just one.
  - (b) If the first non-zero is at the end of the number and there is an odd number of digits to the right of the decimal, then you must add a zero to the end of the number to make the last digit part of a pair of digits.
  - (c) Drop all digits (if any) following this digit pair.
- (3) Replace the remaining pairs of digits.
  - (a) If the pair identified in step 2 is a perfect square (1, 4, 9, 16, 25, 36, 49, 64, or 81), replace the pair with the square root of that number.
  - (b) If the pair identified in step 2 is not a perfect square, identify the largest perfect square that is less than the pair and replace the pair with the square root of that perfect square.
  - (c) For each **pair** of double zero digits between the decimal point and the digit pair identified in step 2, substitute a zero.
- (4) The resulting number is the estimated square root (low).
- (5) Increase the last digit of the estimated square root (low) by 1 to arrive at the estimated square root (high).
- (6) The actual square root will be less than the estimated square root (high) and equal to or greater than the estimated square root (low).

**NOTE:** If the pair of digits identified in step 2 is a perfect square and there were no non-zero digits following the pair in the original number, then the estimated square root (low) is the actual square root.

Estimate the square root of the following numbers.

- a. 0.00004                      b. 0.0004645775

Solution to  
Frame 5-29

No problem was given in  
Frame 5-29.

**FRAME 5-31**

In case you had difficulty with the square roots given in Frame 5-28, the problems are worked in greater detail below.

a. 0.00004

(1) Pair off beginning at the decimal point

(2)  $0.00004. = (00)(00)(40)$

[You must add a zero to the end of the number so that the "4" is part of a pair.]

(3) Largest perfect square not over 40 is 36. The square root of 36 is 6.

(4) (low)  $(00)(00)(40) \rightarrow (0)(0)(6) \rightarrow 0.006$

(5) (high)  $(0)(0)(6+1) \rightarrow 0.007$

b. 0.0004645775

(1) Pair off beginning at the decimal point

(2)  $0004645775. = (00)(04)(64)(57)(75) \rightarrow (00)(04)$

[The digits after the first non-zero pair of digits are dropped.]

(3) Largest perfect square not over 04 is 4. The square root of 4 is 2.

(4) (low)  $(00)(04) \rightarrow (0)(2) \rightarrow 0.02$

(5) (high)  $(0)(2+1) \rightarrow 0.03$

Solution to  
Frame 5-30

a. between 0.006  
and 0.007  
(actual square root is  
0.0063245...)

b. between 0.02 and  
0.03  
(actual square root is  
0.0215540...)

**FRAME 5-32**

Estimate the square root of the following numbers.

a. 0.0036

b. 0.1

c. 0.01

Solution to  
Frame 5-31

No problem was given in  
Frame 5-31.

*Turn Page for Self-Test*

**FRAME 5-33.**

**SELF TEST.** You have completed the section on negative numbers, scientific notation, squares, and square.

If you feel that you need more review, look over the appropriate frames again. Then work the following self-test exercises shown below. The solutions are found on the following page.

1. Work the following problems.

a.  $-3 + -6 =$  \_\_\_\_\_

b.  $-3 - -6 =$  \_\_\_\_\_

c.  $-3 + +6 =$  \_\_\_\_\_

d.  $-3 - +6 =$  \_\_\_\_\_

e.  $-3 \times +6 =$  \_\_\_\_\_

f.  $-3 \times -6 =$  \_\_\_\_\_

g.  $-3 \div -6 =$  \_\_\_\_\_

h.  $+3 \div -6 =$  \_\_\_\_\_

2. Change the following into scientific notation.

a. 16,000.

b. 0.0003

c. 104.3

3. Perform the following operations in scientific notation.

a.  $3 \times 10^4 \times 2.2 \times 10^7$

b.  $3 \times 10^{-4} \times 2.2 \times 10^7$

c.  $3 \times 10^{-4} \times 2.2 \times 10^{-7}$

4. The square root of 4,900 is \_\_\_\_\_

5. The square root of 850,000 is between \_\_\_\_\_ and \_\_\_\_\_.

6. The square root of 0.00005 is between \_\_\_\_\_ and \_\_\_\_\_.

Solution to  
Frame 5-32

a. 0.06 (exact)

.(00) (36)  
(0)(6)

b. between 0.3  
and 0.4

0.1 = 0.10  
. (10)  
. (3) low  
. (3+1) high  
[ $0.3^2 = 0.09$   
 $0.4^2 = 0.16$ ]

c. 0.1 (exact)

.(01)  
. (1)

**Check Your Answers on Next Page**

## SOLUTIONS TO FRAME 5-33 (SELF-TEST)

1.
  - a.  $-9$  (negative + negative = negative sum of absolute values)
  - b.  $+3$  ( $-3 - -6 = -3 + +6$ )
  - c.  $+3$  ( $-3 + +6 = 6 - 3$ )
  - d.  $-9$  ( $-3 - +6 = -3 + -6$ )
  - e.  $-18$  (negative x positive = negative)
  - f.  $18$  (negative x negative = positive)
  - g.  $0.5$  (negative divided by negative = positive)
  - h.  $-0.5$  (positive divided by negative = negative)
  
2.
  - a.  $1.6 \times 10^4$
  - b.  $3 \times 10^{-4}$
  - c.  $1.043 \times 10^2$
  
3.
  - a.  $6.6 \times 10^{11}$
  - b.  $6.6 \times 10^3$
  - c.  $6.6 \times 10^{-11}$
  
4.  $70$  (49)(00)  $\rightarrow$  (7)(0)
  
5.  $900$  and  $1,000$   
(85)(00)(00)  $\rightarrow$  (9)(0)(0) low; (9+1)(0)(0) high  
 $900^2 = 810,000$      $1000^2 = 1,000,000$
  
6.  $0.007$  and  $0.008$  .  
(00)(00)(50)  $\rightarrow$  .(0)(0)(7) low; .(0)(0)(7+1) high  
 $0.007^2 = 0.000049$ ;  $0.008^2 = 0.000064$

**End of Lesson 5**

## APPENDIX

### Metric (SI) Prefixes

<u>Prefix</u>	<u>Meaning</u>	<u>Power of 10</u>	<u>Example</u>
tera-	1,000,000,000,000	$10^{12}$	1 terameters equals one trillion meters
giga-	1,000,000,000	$10^9$	1 gigameter equals one billion meters
mega-	1,000,000 times	$10^6$	1 megameter equals one million meters
kilo-	1,000 times	$10^3$	1 kilometer equals one thousand meters
hecto-	100 times	$10^2$	1 hectometer equals one hundred meters
deca-	10 times	$10^1$	1 decameter equals ten meters
		$10^0$	1 meter equals 1 meter
deci-	1/10	$10^{-1}$	ten decimeters equal 1 meter
centi-	1/100	$10^{-2}$	one hundred centimeters equal 1 meter
milli-	1/1000	$10^{-3}$	one thousand millimeters equal 1 meter
micro-	1/1,000,000	$10^{-6}$	one million micrometers equal 1 meter
nano-	1/1,000,000,000	$10^{-9}$	one billion nanometers equal 1 meter
pico-	1/1,000,000,000,000	$10^{-12}$	one trillion picometers equal 1 meter

### Memory Aids

centi- (1/100) -- a cent is 1/100 of a dollar

deci- (1/10) -- a dime is 1/10 of a dollar

deca- (10) -- a decade is 10 years

hect- or hecto- (100) -- starts with same letter as "hundred"

### Conversion: Length

1 inch (in) = 2.54 centimeters = 25.4 millimeters

1 foot (ft) = 30.48 centimeters = 0.3048 meters

1 yard (yd) = 91.44 centimeters = 0.9144 meters

1 mile (mi) = 1.609 kilometers

1 millimeter (mm) = 0.0394 inches

1 centimeter (cm) = 0.394 inches = 0.0328 feet

1 meter (m) = 1.094 yards = 3.28 feet = 39.4 inches

1 kilometer (km) = 0.621 (about 5/8) miles = 1094 yards = 3281 feet

### **Conversion: Area**

1 square inch (in<sup>2</sup>) = 6.452 square centimeters  
1 square foot (ft<sup>2</sup>) = 929.03 square centimeters = 0.092903 square meters  
1 square yard (yd<sup>2</sup>) = 0.836 square meters  
1 acre = 4,047 square meters = 40.47 ares = 0.4047 hectare  
1 square mile = 2.590 square kilometers = 259 hectare

1 square centimeter (cm<sup>2</sup>) = 0.155 square inches  
1 square meter (m<sup>2</sup>) = 10.764 square feet = 1.196 square yards  
1 are = 100 m<sup>2</sup> = 0.02471 acres = 119.599 square yards  
1 hectare (ha) = 100 ares = 10,000 m<sup>2</sup> = 2.471 acres  
1 square kilometers (km<sup>2</sup>) = 0.386 square miles

### **Conversion: Volume (Capacity)**

1 cubic inch (in<sup>3</sup>) = 16.387 cubic centimeters (milliliters)  
1 cubic foot (ft<sup>3</sup>) = 28.4 cubic decimeters (liters)  
1 cubic yard (yd<sup>3</sup>) = 0.764 cubic meters = 764 liters

1 cubic centimeter (cc) = 1 milliliter = 0.061 cubic inches  
1 cubic decimeter = 1 liter = 61 cubic inches = 0.035 cubic feet  
1 cubic meter (m<sup>3</sup>) = 35.314 cubic feet = 1.308 cubic yards

### **Conversion: Liquid Volume (Capacity)**

1 teaspoon (tsp) = 4.93 milliliters  
1 tablespoon (Tbsp) = 14.79 milliliters  
1 fluid ounce (fl oz) = 29.57 milliliters  
1 cup (c) = 236.6 milliliters  
1 pint (pt) = 473.2 milliliters = 0.4732 liters  
1 quart (qt) = 946.4 milliliters = 0.9464 liters  
1 gallon (gal) = 3.785 liters

1 milliliter (mL) = 0.0338 fluid ounce  
1 liter (L) = 0.264 gallons = 1.057 quarts = 2.11 pints = 33.8 fluid ounces

## Conversion: Weight/Mass

### Avoirdupois

1 grain (gr) = 64.7989 milligrams = 0.0648 grams  
1 ounce (oz) = 28.3495 grams  
1 pound (lb.) = 16 ounces = 453.6 grams = 0.4536 kilograms  
1 hundredweight = 45.36 kilograms = 0.04536 metric tons  
1 ton (2,000 pounds) = 907 kilograms = 0.907 metric tons

1 milligram (mg) = 0.01543 grains  
1 gram (g) = 15.4324 grains = 0.03528 ounces  
1 kilogram (kg) = 2.20462 pounds = 35.28 ounces  
1 metric ton (t) = 1194.62 pounds = 1.102 short tons

### Apothecaries'

1 grain = 64.7989 milligrams  
1 scruple (20 grains) = 1.3 grams  
1 dram (3 scruples) = 3.9 grams  
1 ounce (8 drams) = 31.1 grams  
1 pound (12 ounces) = 373 grams

### Troy

1 grain = 64.7989 milligrams  
1 ounce (480 grains) = 31.1 grams  
1 pound (12 ounces) = 373 grams

## Conversion: Temperature

### Celsius to Fahrenheit (multiple, then add)

$$^{\circ}\text{F} = (^{\circ}\text{C} \times 1.8) + 32^{\circ} \quad \text{or} \quad ^{\circ}\text{F} = 9/5 ^{\circ}\text{C} + 32^{\circ}$$

### Fahrenheit to Celsius (subtract, then multiply)

$$^{\circ}\text{C} = \frac{5 (^{\circ}\text{F} - 32^{\circ})}{9} \quad \text{or} \quad ^{\circ}\text{C} = 0.555556 (^{\circ}\text{F} - 32^{\circ})$$

### Celsius to kelvin

$$\text{K} = ^{\circ}\text{C} + 273.15$$

## Quick Reference

	kelvin	Celsius	Fahrenheit
Absolute zero	0	- 273.15	- 459.67 °F
Water freezes	273.15	0	32
Water boils	373.15	100	212

## **Rules for Adding, Subtracting, Multiplying, and Dividing Positive and Negative Numbers**

### Addition of Two Positive Numbers

Add their absolute values and place a positive symbol (or no symbol) in front of the sum.

### Addition of Two Negative Numbers

Add their absolute values and place a negative symbol in front of the sum.

### Addition of a Positive Number and a Negative Number

Subtract the smaller absolute value from the larger absolute value, then place the original sign of the larger absolute value in front of the difference.

### Subtraction

Change the sign of the subtrahend (second or bottom number), then add the two numbers according to the rules for addition given above.

### Multiplication/Division of Two Numbers

Multiply/divide the absolute values as you would normally. If both original numbers have the same sign (both positive or both negative), make the product/quotient positive. If the original numbers had different signs (one positive and one negative), make the product/quotient negative.

### Multiplication of More Than Two Numbers

Multiply the absolute values as you would normally. Count the number of original numbers that were negative. If that number is even (0, 2, 4, 6, etc.), make the product positive. If that number is odd (1, 3, 5, 7, etc.), make the product negative.

## MULTIPLICATION TABLE

(Factors are in bold. The product is located at the intersection of the factors.)

<b>X</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0</b>	0	0	0	0	0	0	0	0	0	0
<b>1</b>	0	1	2	3	4	5	6	7	8	9
<b>2</b>	0	2	4	6	8	10	12	14	16	18
<b>3</b>	0	3	6	9	12	15	18	21	24	27
<b>4</b>	0	4	8	12	16	20	24	28	32	36
<b>5</b>	0	5	10	15	20	25	30	35	40	45
<b>6</b>	0	6	12	18	24	30	36	42	48	54
<b>7</b>	0	7	14	21	28	35	42	49	56	63
<b>8</b>	0	8	16	24	32	40	48	56	64	72
<b>9</b>	0	9	18	27	36	45	54	63	72	81

# COMMENT SHEET

**SUBCOURSE MD0900 Basic Mathematics**

**EDITION 100**

Your comments about this subcourse are valuable and aid the writers in refining the subcourse and making it more usable. Please enter your comments in the space provided. ENCLOSE THIS FORM (OR A COPY) WITH YOUR ANSWER SHEET **ONLY** IF YOU HAVE COMMENTS ABOUT THIS SUBCOURSE.

**FOR A WRITTEN REPLY, WRITE A SEPARATE LETTER AND INCLUDE SOCIAL SECURITY NUMBER, RETURN ADDRESS (and e-mail address, if possible), SUBCOURSE NUMBER AND EDITION, AND PARAGRAPH/EXERCISE/EXAMINATION ITEM NUMBER.**

## PLEASE COMPLETE THE FOLLOWING ITEMS:

(Use the reverse side of this sheet, if necessary.)

1. List any terms that were not defined properly.

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2. List any errors.

paragraph      error      correction

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3. List any suggestions you have to improve this subcourse.

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4. Student Information (optional)

Name/Rank \_\_\_\_\_  
SSN \_\_\_\_\_  
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Telephone number (DSN) \_\_\_\_\_  
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### PRIVACY ACT STATEMENT (AUTHORITY: 10USC3012(B) AND (G))

**PURPOSE:** To provide Army Correspondence Course Program students a means to submit inquiries and comments.

**USES:** To locate and make necessary change to student records.

**DISCLOSURE: VOLUNTARY.** Failure to submit SSN will prevent subcourse authors at service school from accessing student records and responding to inquiries requiring such follow-ups.